Instabilities in Dense Neutrino Systems

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Variables: ν density (matrix) operators in flavor representation

$$\rho_{i,j}(\mathbf{p}) = a_i(\mathbf{p})^\dagger \, a_j(\mathbf{p})$$
$$\bar{\rho}_{i,j}(\mathbf{p}) = \bar{a}_j(\mathbf{p})^\dagger \, \bar{a}_i(\mathbf{p})\,,$$

$\nu - \nu$ interaction (dead forward)

$$H_{\nu\nu}(\rho) = \frac{\sqrt{2}G_F}{V} \sum_{\mathbf{p},\mathbf{q}} \; \sum_{\{i,j\}=e,x} [1 - \cos(\theta_{\mathbf{p},\mathbf{q}})]$$
$$\times \Big[\Big(\rho_{i,j}(\mathbf{p}) - \bar{\rho}_{i,j}(\mathbf{p})\Big) \Big(\rho_{j,i}(\mathbf{q}) - \bar{\rho}_{j,i}(\mathbf{q})\Big)$$
$$+ \Big(\rho_{i,i}(\mathbf{p}) - \bar{\rho}_{i,i}(\mathbf{p})\Big) \Big(\rho_{j,j}(\mathbf{q}) - \bar{\rho}_{j,j}(\mathbf{q})\Big) \Big]\,,$$

Add ν oscillation term, H_{osc}

→ Non-linear equations for $\rho_{i,j}$ (p)

$$\frac{d}{dt}\rho_{i,j}(\mathbf{p}) = F_{i,j}(\rho' s)$$

To solve: make N_B angular bins. Get $4N_B$ coupled nonlinear equations.

Usual initial conditions: flavor-diagonal

$$\rho_{i,j}(\mathbf{p}, 0) = 0 \quad \text{when } i \neq j$$

Many papers by many authors experiment with simulations focused on the region outside of the ν surface

These find:

A plethora of possible exotic phenomena.

Quite parameter dependent:

1. Major action only for inverted hierarchy

2. High electron density can quench

The work I report here

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differs in three respects:

- The application:

  Neutrino flavor mixing in the region below the surface of last scattering (ν surface)

- The nature of the instability-inducing mixing:

  Different, but related in a defined way

  (and perhaps more robust)

- Sensitivity to parameters:

  Insensitive to “normal” vs. “inverted” distinction, and to electron density

To understand this instability:

A. Turn off oscillations.

(Then with flavor diagonal initials nothing happens.)

B. Linearize in the off-diagonal densities.

C. Do standard linear stability analysis.

Depending on the detailed flavor density vs. angle (i.e. bin) dependence we can get:

- Just oscillatory modes.
- In addition, a pair of zero modes
- A growing mode

Growing mode case

- The time constant is of order $(G_F\ n_B)^{-1}$.
- $(G_F\ n_B)^{-1} = 10^{-3}$ cm at a density of 10^{11} g cm^{-3}.

So the least bit of initial flavor mixing might induce
very rapid flavor exchanges.

When do we get this instability?

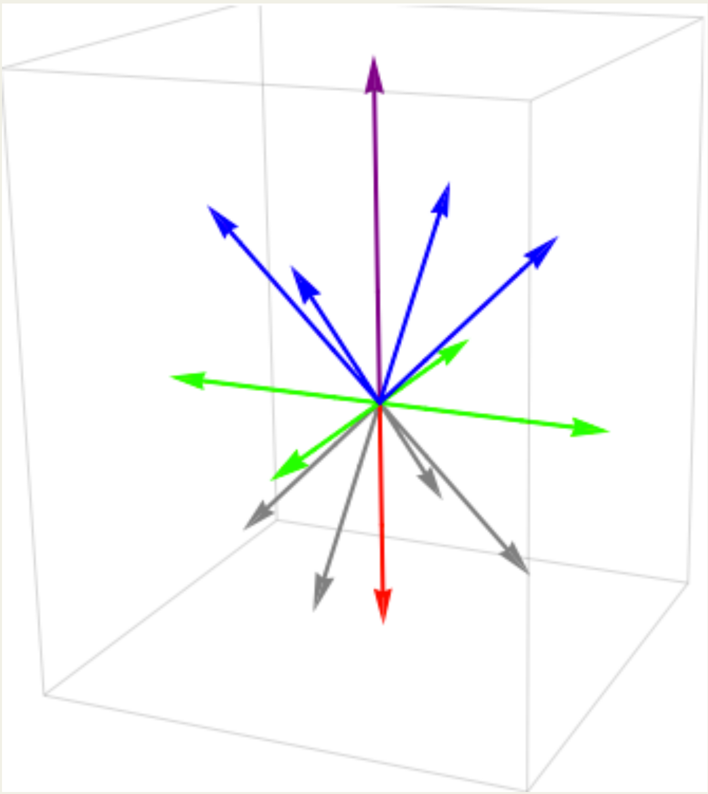
Answer: when angular distributions are "irregular" enough.

In neutrino-sphere region ν momenta are biased upward

(ν_μ and ν_τ more so than ν_e)

Start with cylindrical symmetry, as shown in the bins :

up



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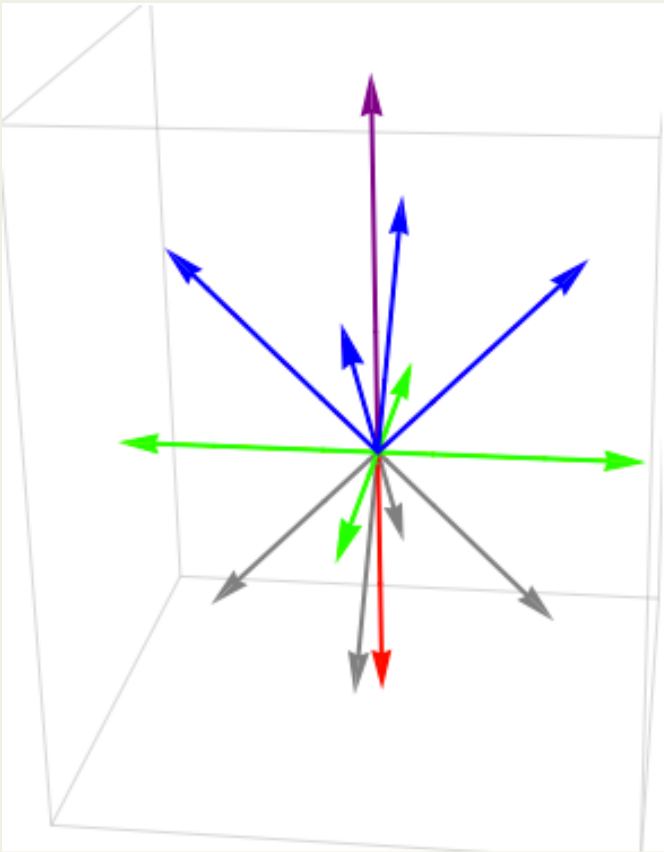

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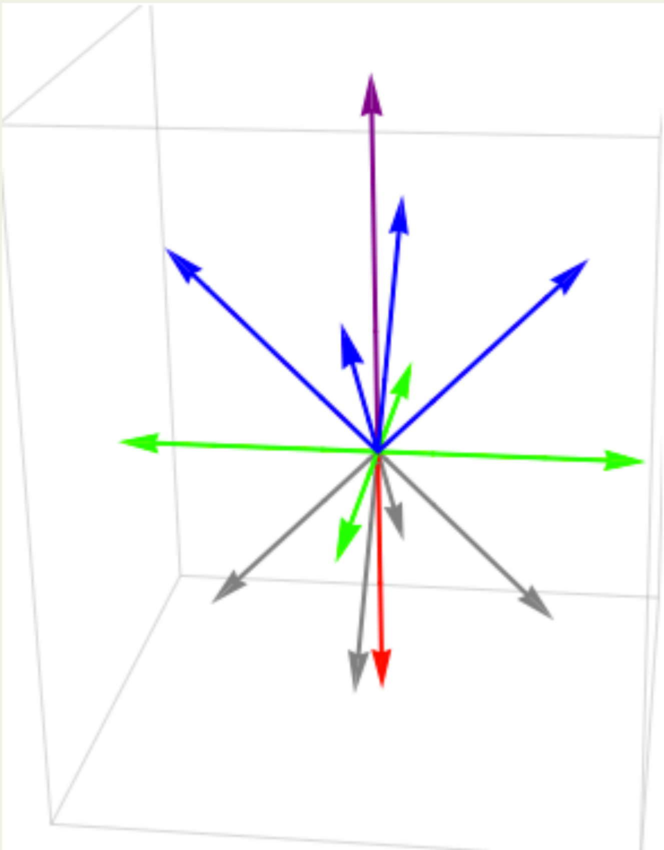
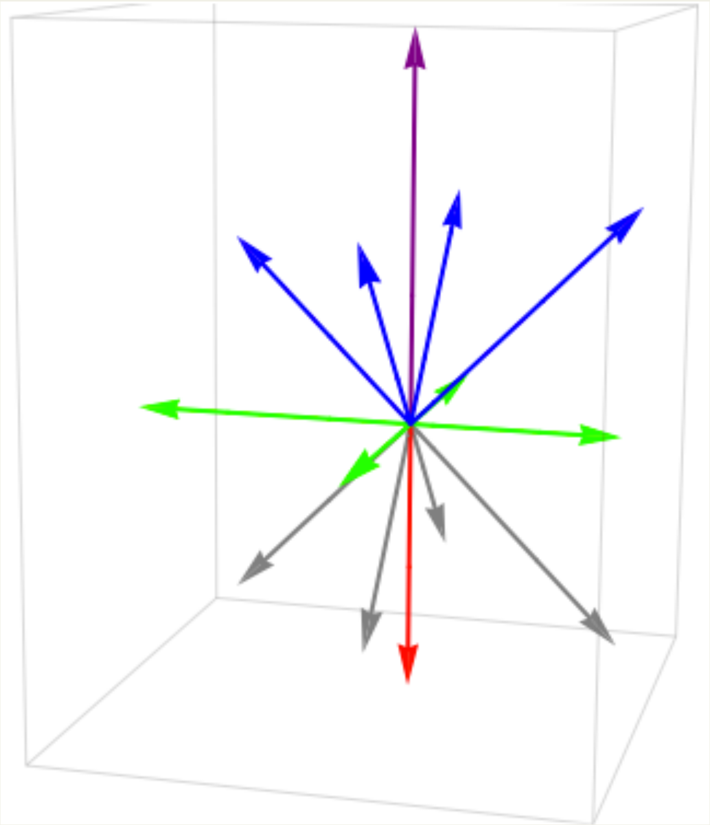


Get zero modes---no growing modes

- Add independent random 5% length variations on each vector-
- Breaking symmetries of rotation around z axis

up



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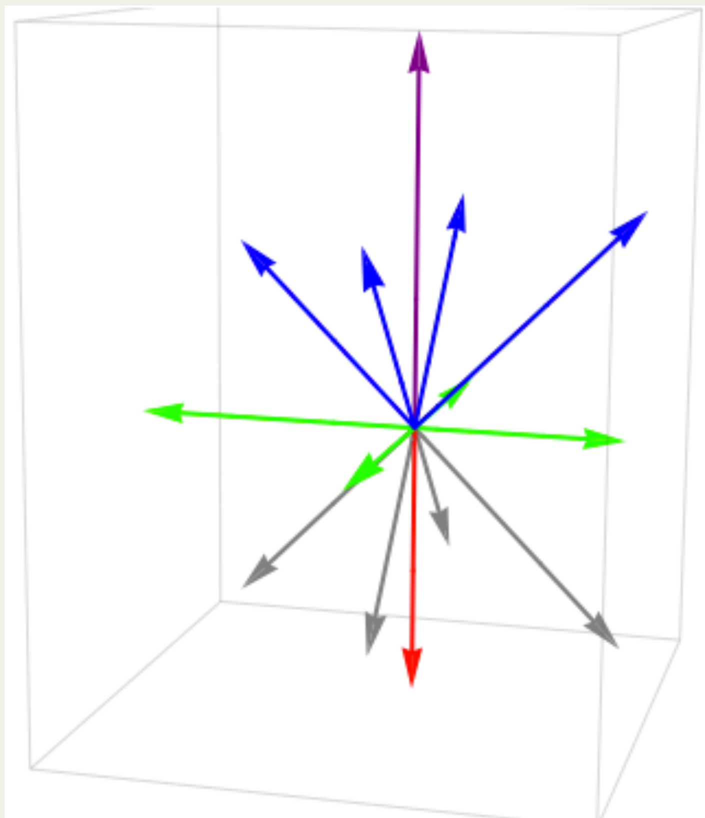


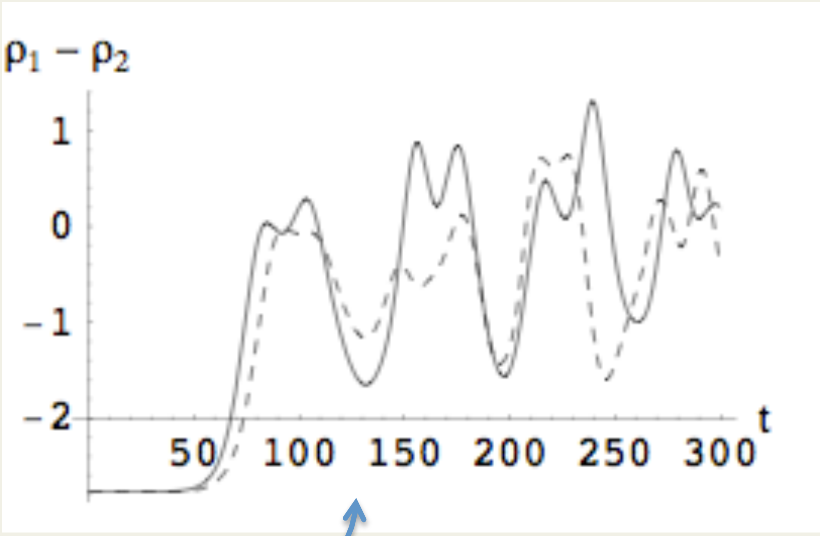
Now: 50% of the time we get a growing mode

- Taking such a growing mode case, and adding in ν oscillation terms—

- Do the complete simulation (2 flavor)

ρ_1 = number of ν_e momenta in the upward hemisphere

ρ_2 = number of ν_x momenta in the upward hemisphere



solid = normal

dashed= inverted

. 1 cm

- Shows large exchange of flavor between ν_x beam , originally strongly biased upward, and ν_e beam, originally more isotropic.

Sources of input angular distributions

• The outward biases come from solution of the radiative transfer equations near a surface in a non-lumpy spherically symmetric interior.

•They increase steadily going upwards through the surface regions.

NOTE: At a given depth ν_x is more biased outward than ν_e .

• The azimuthal irregularities come from scattering in the lumpy medium.

(reminding one of the manner in which matter inhomogeneities are turned into radiation anisotropies in the big bang before recombination.)

Details:

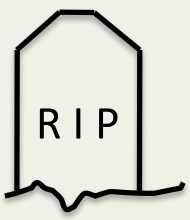
- Virtually the same result for normal and inverted hierarchies.
- Neutrino-electron interaction does not dampen phenomenon.
- Can be damped by too big an excess of ν_e over anti $-\nu_e$
  (tolerates a 20% difference)

Result

Possible complete flavor mixing of outgoing stream

(i.e. All three flavors with the same energy spectrum)

Would this matter?



1. Could affect the dynamics of the SN, through increased heating from electron neutrinos with jacked up energies.

2. Would affect R process nucleosynthesis.

3. Would drastically affect the neutrino pulse as observed in ν observatory.

But, in 2035 will anybody care?

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Have done eigenvalue calculations (in the linearized equations)
for 26 rays. Find same patterns of growing mode likelihood
as for the 14 in the pictures.

But then doing the full nonlinear evolution
(now with oscillation terms, as well)
was too big for me to compute at the time.

However:

Complex eigenvalue [or "growing mode"] → rapid flavor exchange

is a generic property

Nevertheless, it would be good if one of the groups with more people and computing resources would look into cases that break cylindrical symmetry.

Some comments on the above-the-neutrino-surface-simulations in the literature:

………insofar as they are related to multi-angle instabilities

- As far as I know all have used an angular distribution with cylindrical symmetry with respect to a radius vector from the star.

- In our modes analysis when there is cylindrical symmetry, no matter how peaked in the z direction, we get zero modes but no growing modes.

- Return for a moment to idealized conditions in which matter density and ν angular distributions do not change much in the region of the simulation------then:

- With cylindrical symmetry and just $\nu - \nu$ plus vacuum oscillation parameter λ in H, we get , at the least a waiting period time t_w of order

$$t_w = (G_F\, n_\nu\, \lambda)^{-1/2}$$

(The geometric mean of the very fast and the pure oscillation time scales)

followed by a sudden flavor-switching transition.

This is too simple a picture, for the cylindrical case,

and above the ν−surface

Complications:

1. Electron density reduces the mixing induced by λ , extending the waiting time.

2. In the application above the ν surface, parameters change appreciably along the ν path.

- ν angular distribution (narrows with distance from star)
- ν density
- electron density

But I think that qualitative aspects of the simulations that gave the waiting time

$$t_w = (G_F\, n_\nu\, \lambda)^{-1/2}$$

are relevant to the phenomena that have been found in above-the-ν–surface simulations

The interpretation would be that the oscillation term changes the parameters that enter linear stability analysis so that after some time there is an appreciably imaginary eigenvalue, and consequent exponential growth of mixing.

Conclusion:

There may be some physics here.