Equation of State and neutrino signals in supernova

• New astrophysical equations of state (EOS):
– Virial expansion with nucleons, alphas + thousands of heavy nuclei at low densities.
– Extensive relativistic mean field calculations at high densities.

• Late time neutrino luminosity and EOS:
– density dependence of symmetry energy significantly effects the time over which convection operates.

• Spin response of neutron matter:
– QMC sum rules constrained spin response: peak ~ 40 MeV.
– enhanced bremsstrahlung rate compared to OPE: mu and tau neutrino productions.

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"HAvSE: HAmburg neutrinos from Supernova Explosions"
Hamburg, Germany, Jul. 19 – 23, 2011

Collaborators

EOS: Chuck Horowitz, Evan O'Connor

PNS cooling: Luke Roberts, Sanjay Reddy, Vincenzo Cirigliano, Jose Pons, Stan Woosley

Spin response: Joe Carlson, Stefano Gandolfi, Sanjay Reddy

New Astrophysical Equations of State

- For simulations of supernovae, neutron star mergers, black hole formation...
- Cover density n, temperature T, and proton fraction Yp over large range (calculated at 180,000 points) $0<Yp<0.56$, $0<T<80$ MeV, $10^{-8}<n<1.6$ fm^{-3}.
- Almost all realistic SN simulations use
  - J.M. Lattimer, F.D. Swesty – liquid droplet model + skyrme force
  - H. Shen, H. Toki, K. Oyamatsu, K. Sumiyoshi - relativistic mean field model in Thomas-Fermi approximation and variational cal.
- Recently, nuclear statistical model + uniform matter at high density
  - M. Hempel, J. Schaffner-Bielich

- Our EOSs use extensive relativistic mean field calculations at high densities and virial + nuclear statistical model at low densities.
  EOS tables at http://cecelia.physics.indiana.edu/gang_shen_eos/

Virial + Nuclear Statistical Model

- At low densities consider nucleons + alphas + thousands of heavy nuclei (A>11).
- Expand pressure in powers of z :

$$\frac{\log \mathcal{Q}}{V} = \frac{P}{T} = \frac{2}{\lambda_n^3}[z_n + z_p + (z_p^2 + z_n^2)b_n + 2z_p z_n b_{pn}] + \frac{1}{\lambda_\alpha^3}[z_\alpha + z_\alpha^2 b_\alpha + 2z_\alpha(z_n + z_p)b_{\alpha n}] + \sum_i \frac{1}{\lambda_i^3} z_i \Omega_i$$

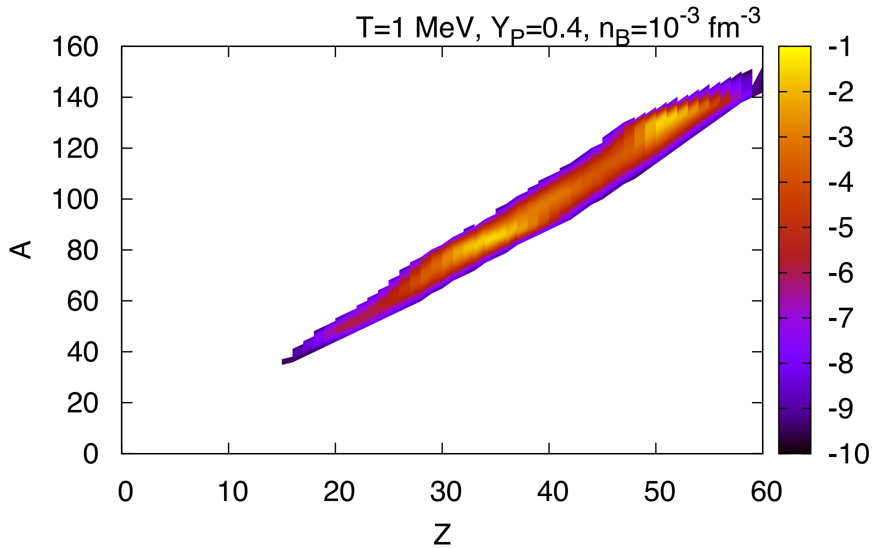
- 2nd virial coeff. b2(T) from elastic scattering phase shifts

– b_n for neutron matter, – b_{np} for symmetric nuclear matter,
– b_α for alpha system, – $b_{\alpha n}$ for interaction between alpha and N.

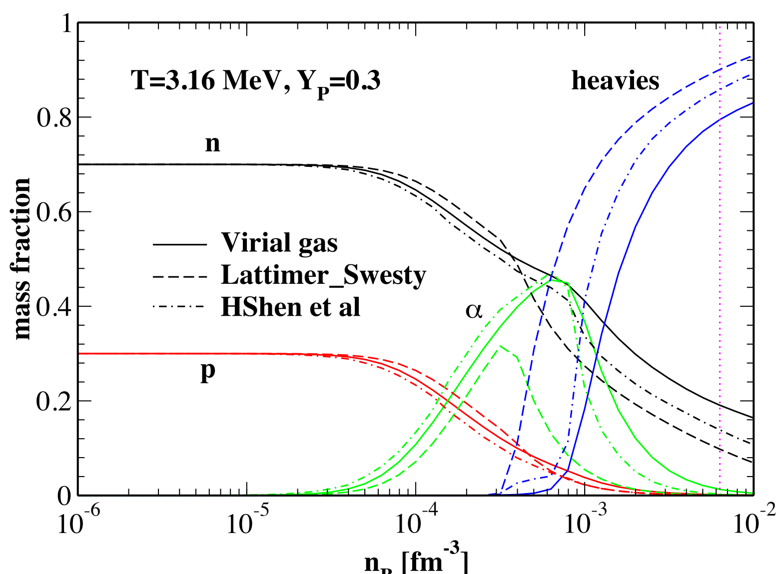
- Heavy nuclei: $z_i = \exp(\mu_i + E_i)/T = z_p^Z z_n^N e^{E_i/T}$

Ω i=partition function for heavy nuclei, *Ei is binding E.*

Composition: nucleons, alphas, heavy nuclei



- The previous two EOS used single average nucleus approximation.
- In our EOS, mass distribution of heavy nuclei is often multi peaked Gaussian distribution.
- This may greatly influence the infall phase of supernova, particularly lepton capture rate: eg, Juodagalvis et al, 2010.



<-- Virial+ NSE | RMF -->

- Attractive interactions between alphas and nucleons increase mass fraction of alpha
- The change in composition may influence the position of neutrino sphere and neutrino spectrum,

  eg, Arcones et al, 2008

Relativistic Mean Field Calculations

attractive — repulsive — Iso-vector

$$
\begin{aligned}
\mathscr{L}_{\rm int} = {} & \bar{\psi}\left[g_{\rm s}\phi - \left(g_{\rm v}V_\mu + \frac{g_\rho}{2}\boldsymbol{\tau}\cdot\mathbf{b}_\mu + \frac{e}{2}(1+\tau_3)A_\mu\right)\gamma^\mu\right]\psi \\
& - \frac{\kappa}{3!}(g_{\rm s}\phi)^3 - \frac{\lambda}{4!}(g_{\rm s}\phi)^4 + \frac{\zeta}{4!}g_{\rm v}^4(V_\mu V^\mu)^2 \\
& + \Lambda_{\rm v} g_\rho^2 \mathbf{b}_\mu\cdot\mathbf{b}^\mu g_{\rm v}^2 V_\nu V^\nu . \qquad (1)
\end{aligned}
$$

κ , λ : incompressibility
ζ : stiffness
Λ_v : symmetry energy

- at higher densities, use rela. mean field model
- lattice in crust: spherical Wigner Seitz boundary conditions: given T, n_B, Y_p, minimize free E wrt cell radius R_s.
- Finite temperature T calc. with very many levels.
- Repeat for large numbers of n_B, T, Y_p. – 200,000 CPU hrs
- Now have three EOSs available

– Stiff: based on NL3 interaction, maximum NS mass is 2.7 M_{sun}. --> NL3
– Soft: based on FSUgold, maximum mass is 1.7 M_{sun}. -->FSU1.7
– Medium: modify FSUgold at high densities, maximum mass is 2.1 M_{sun}. --> FSU2.1
– Future: IU-FSU like (with smaller nuclear symmetry energy)

Equation of State Tables

- ~2 million lines with 16 numbers per line. Each line has:
  - 9 thermodynamic quantities:

    $T, Y_p, n, F, P, S, \mu_n, \mu_p, \mu_e$
  - 7 quantities provide composition information for neutrino interactions: x_n, x_p, x_a, x_h, <A>, <Z>, M* (mass fractions for neutrons, protons, alphas, and heavy nuclei, average A, Z of heavy nucleus, effective mass)
- Mass fraction of alpha particles, etc., only has meaning in association with routine to calculate neutrino interactions.

Shen, Horowitz, O'Connor (2011).
Shen, Horowitz, Teige (2011).

Nuclear Symmetry Energy

$$E(n_B, x_p) = E(n_B, x_p = 1/2) + E_{\rm sym}(n_B)\delta^2 + \cdots$$

$$\delta = (1 - 2x_p)$$

- E_{sym} describes how energy changes with p-n asymmetry
- Pressure ~ E'_{sym}, or large $E'_{sym}(n_0)$ – stiff EOS (in p-n asymmetry)
- Large uncertainties in E_{sym} and E'_{sym} - reflected in diff. EOS
- E'_{sym} ~ neutron radius of neutron rich nucleus – Pb208
- E'_{sym} ~ neutron star radius
- E'_{sym} ~ Proto-Neutron Star (PNS) cooling

Models	NL3	FSUGold	IU-FSU
$n_0E'_{sym}(n_0)$ [MeV]	39.4	20.2	15.7

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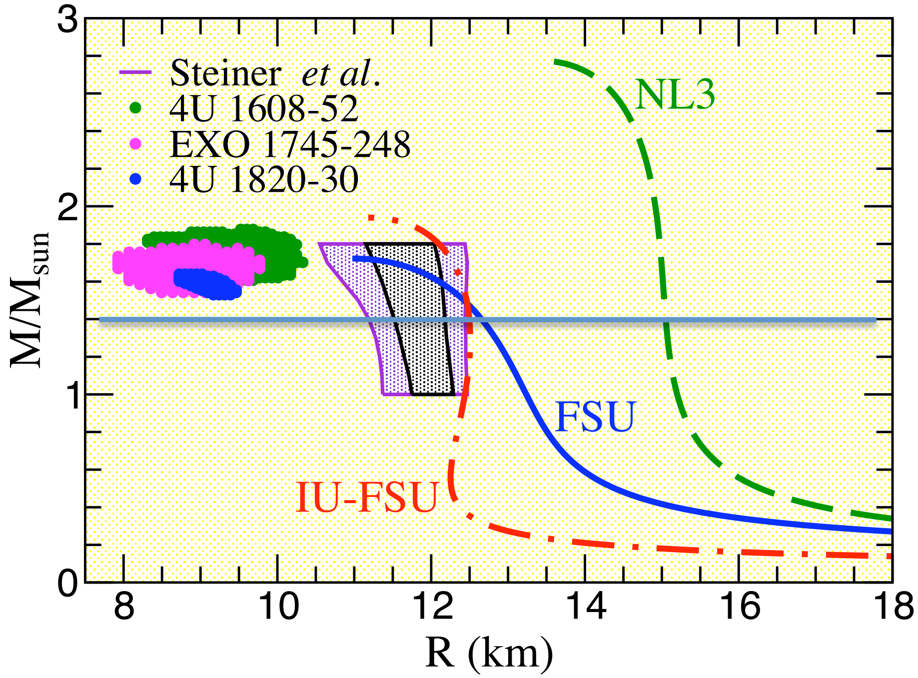
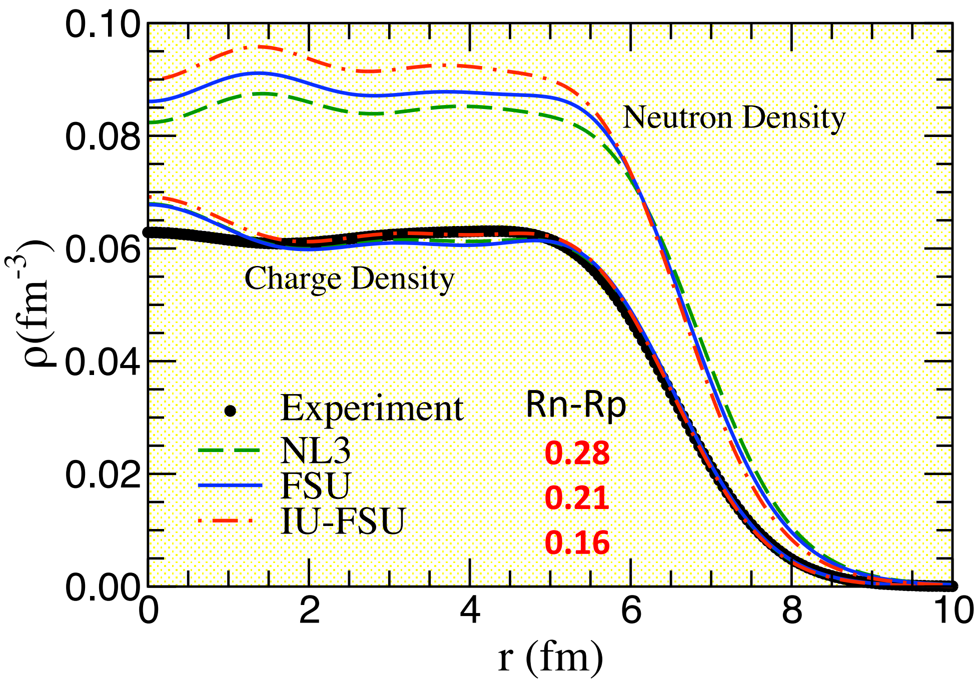
- Neutron star radius and neutron radius in Pb208





PREX: 1% in Rn

A larger $E'_{sym}(n_0)$ indicates a bigger radius for 1.4 solar mass neutron star and a bigger neutron radius in 208Pb.

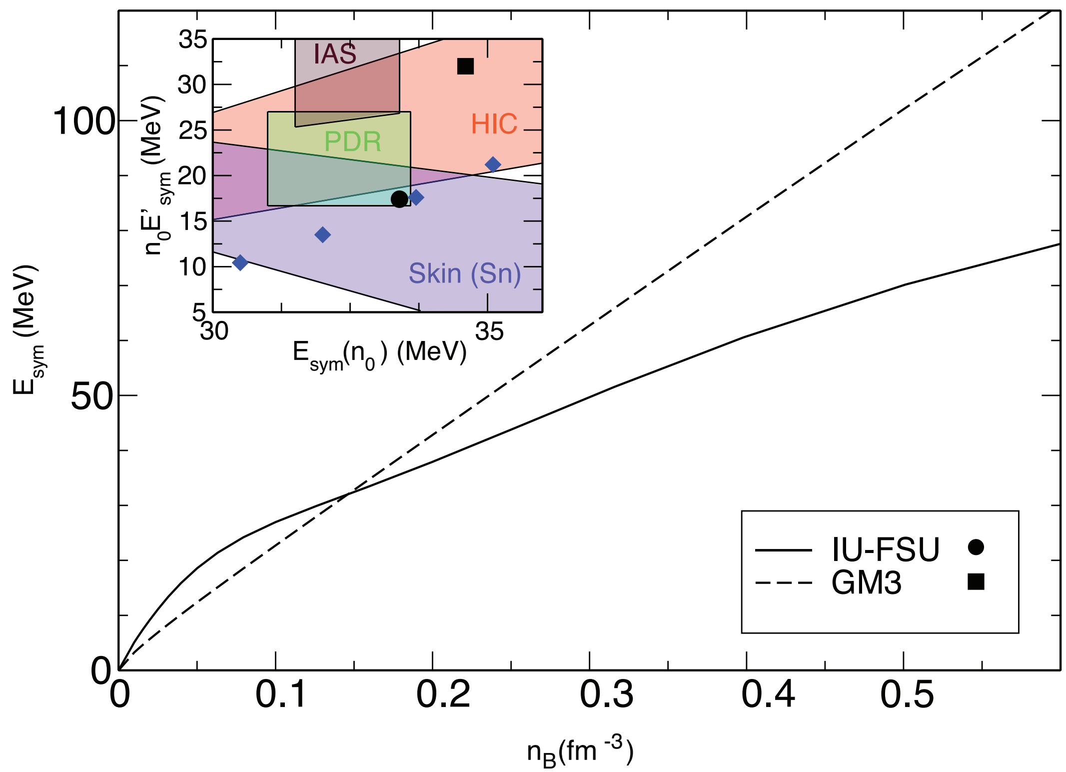
Fattoyev, Horowitz, Piekarewicz, Shen (2010)

Modeling PNS cooling with different EOS

$$\mathscr{L}_{\rm int} = \bar{\psi}\left[g_{\rm s}\phi - \left(g_{\rm v}V_\mu + \frac{g_\rho}{2}\boldsymbol{\tau}\cdot\mathbf{b}_\mu + \frac{e}{2}(1+\tau_3)A_\mu\right)\gamma^\mu\right]\psi$$
$$- \frac{\kappa}{3!}(g_{\rm s}\phi)^3 - \frac{\lambda}{4!}(g_{\rm s}\phi)^4 + \frac{\zeta}{4!}g_{\rm v}^4(V_\mu V^\mu)^2 + \Lambda_{\rm v} g_\rho^2\,\mathbf{b}_\mu\cdot\mathbf{b}^\mu g_{\rm v}^2 V_\nu V^\nu$$

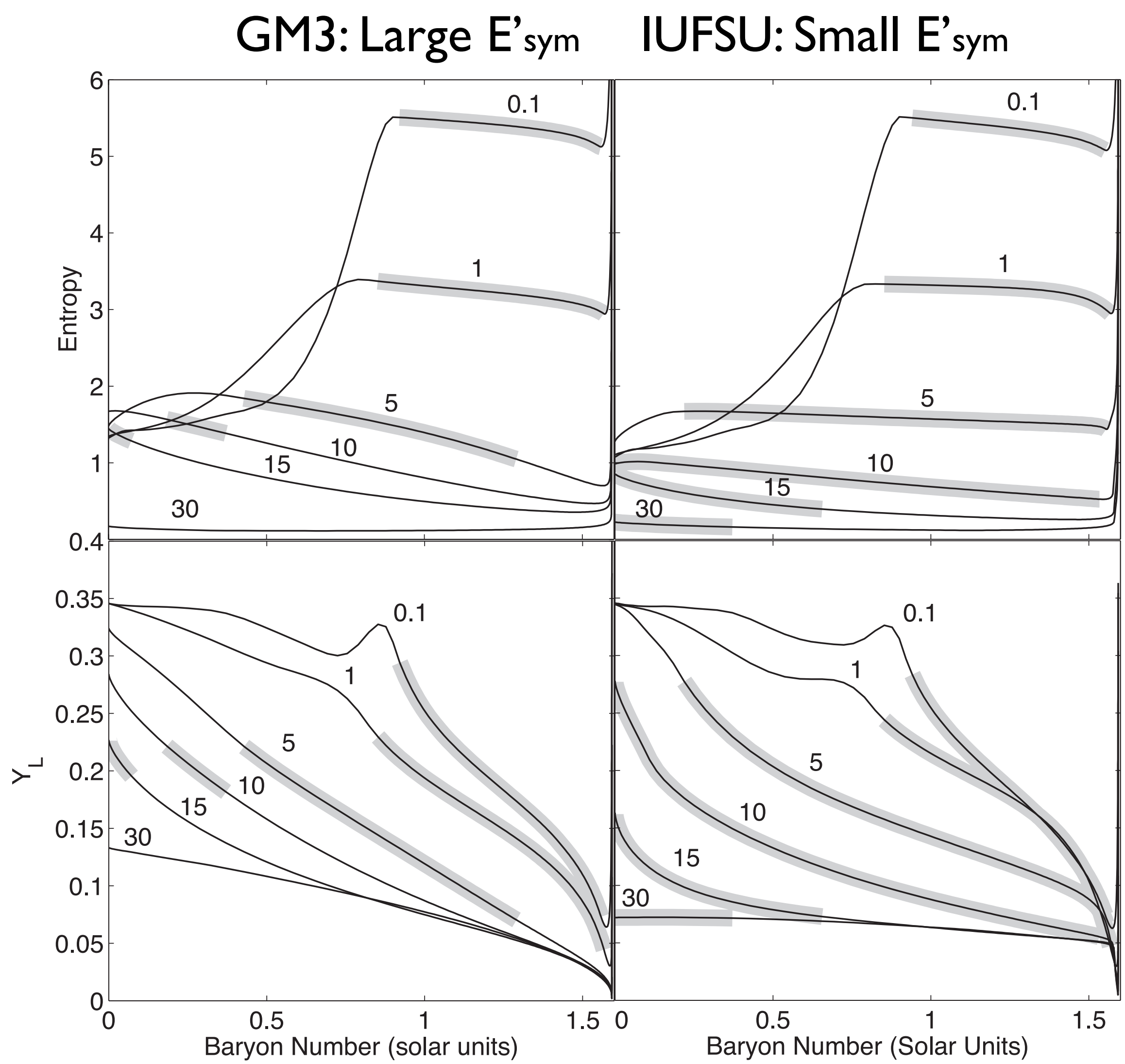
$\Lambda_{\rm v}$: to tune symmetry energy

- Can assume uniform n, p, e matter.
- 2 EOS + diffusion + convection + neutrino opacity (RPA)



Newly born neutron star: An intense neutrino source

- Proto-neutron star evolution time scale is set by neutrino diffusion and convection.
- Grey – unstable convection.
- It is imprinted on the temporal structure of the neutrino.



Roberts, Shen, Cirigliano, Pons, Reddy, Woosley (2011)

Convection is driven by unstable gradients in entropy and lepton number. Instability sets in when $\omega^2 > 0$:

$$\omega^2 = -\frac{g}{\gamma_{n_B}} \left(\gamma_s \nabla \ln(s) + \gamma_{Y_L} \nabla \ln(Y_L) \right)$$

<0 <0

$$\gamma_{n_B} = \left(\frac{\partial \ln P}{\partial \ln n_B} \right)_{s, Y_L} \quad \gamma_s = \left(\frac{\partial \ln P}{\partial \ln s} \right)_{n_B, Y_L} \quad \gamma_{Y_L} = \left(\frac{\partial \ln P}{\partial \ln Y_L} \right)_{n_B, s}$$

>0 >0

Density dependence of nuclear symmetry is key to Understand composition driven convective instability:

$$\left(\frac{\partial P}{\partial Y_L} \right)_{n_B} \simeq n_B^{4/3} Y_e^{1/3} - 4 n_B^2 E'_{\rm sym} (1 - 2Y_e)$$

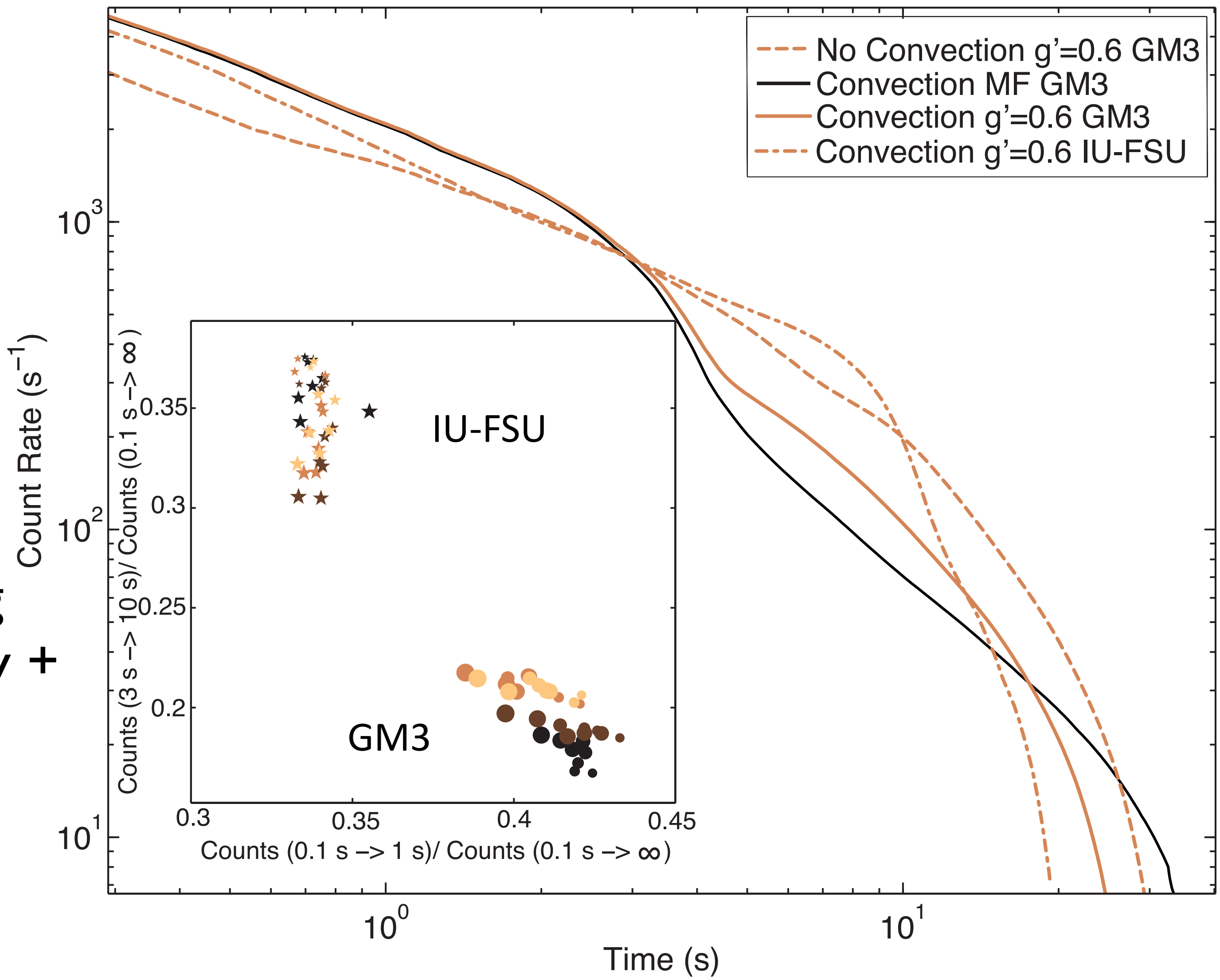
Large E'_{sym} stabilizes convection at late time

Roberts, Shen, Cirigliano, Pons, Reddy, Woosley (2011)

Observable signatures of convective transport

- Count rate in Super-Kamiokande for galactic supernova at 10 kpc.

- Varying mixing length + opacity + PNS masses do not change the conclusion.



Roberts, Shen, Cirigliano, Pons, Reddy, Woosley (2011)

Summary

- New astrophysical equations of state available:

  http://cecelia.physics.indiana.edu/gang_shen_eos/

  – Virial expansion with nucleons, alphas + thousands of heavy nuclei at low densities.

  – Extensive relativistic mean field calculations at high densities.

- Late time neutrino luminosity is correlated with density dependence of symmetry energy – significantly effects the time over which convection operates.

Thank you !

Spin response of neutron matter

- Current conservation: density response suppressed at long wavelength limit,

Density fluc.-> 0, if q -> 0

$$\frac{\partial \mathcal{O}(r)}{\partial t} + \nabla \cdot \mathbf{j}(\mathbf{r}) = 0 \qquad (\mathcal{O}^+_{\mathbf{q}})_{j0} = \frac{\mathbf{q} \cdot (\mathbf{j}^+_{\mathbf{q}})_{j0}}{\omega_{j0}}$$

  response restricted in $q_0 < q$ region.

- Axial vector current is not conserved:

  - spin response contributes even in $q_0 > q$ region, via excitation of multi-pairs.

- Response function should satisfy sum rules
- Important for
  - neutrino bremsstrahlung
  - inelastic neutrino scattering

Studied by Sigl ‘96, Hannestad & Raffelt ‘98, Lykasov, Pethick, & Schwenk ’08, …

Spin response sum rules

$$S_A(\omega, \mathbf{q}) = \frac{4}{3n}\frac{1}{2\pi}\int_{-\infty}^{\infty} dt e^{i\omega t}\langle \mathbf{s}(t,\mathbf{q})\mathbf{s}(t,-\mathbf{q})\rangle$$

Fermi gas

Spin susceptibility $S_{-1} = \int_0^{\infty} \frac{S_A(\omega,0)}{\omega} d\omega, \ \chi = 2nS_{-1}$ $\chi_{FG} = mk_F/\pi^2$

Pair distribution function $S_0 = \int_{-\infty}^{\infty} S_A(\omega,\mathbf{q}) d\omega = 1 + \frac{4}{3N}\sum_{i\neq j}^{N}\langle e^{-i\mathbf{q}\cdot(\mathbf{r_i}-\mathbf{r_j})}\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j\rangle,$

Energy-weighted sum rule $S_{+1} = \int_{-\infty}^{\infty} S_A(\omega,\mathbf{q})\omega d\omega = -\frac{4}{3N}\langle [H, \boldsymbol{s}(\mathbf{q})]\cdot \boldsymbol{s}(-\mathbf{q})\rangle,$

T=0, long wavelength limit q=0: Auxiliary Field Diffusion Monte Carlo

$n = 0.16$ fm^{-3}	S_{-1} [MeV^{-1}]	S_0	S_{+1} [MeV]
AFDMC	0.0045	0.20	11.1

$\chi/\chi_{FG} \sim 0.35$

$S_1/S_0 \sim 56$ MeV → Shape of response function

$S_0/S_{-1} \sim 44$ MeV

Shen, Gandolfi, Reddy, Carlson(2011)

Ansatz for spin response

- Long wave length limit, the spin response function can be written as

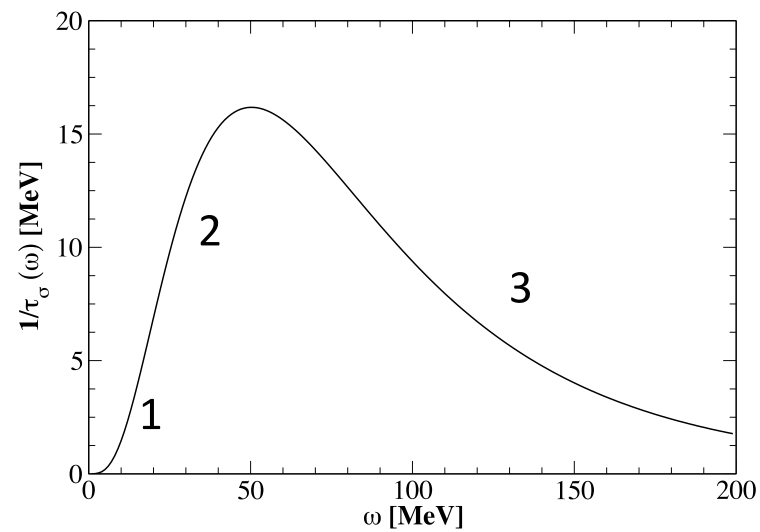
$$S_A(\omega) \;=\; \frac{N(0)}{n\pi}\frac{\omega\tau_\sigma}{(1+G_0)^2+(\omega\tau_\sigma)^2}.$$

- τ_σ - spin relaxation time for quasi-particle, is infinity at both low frequency and high frequency

$$\frac{1}{\tau_\sigma} \sim \mathrm{Tr}\left[A_{\sigma_1,\sigma_2}(1,2\to 3,4)\vec{\sigma_1}[\vec{\sigma_1}+\vec{\sigma_2},A_{\sigma_1,\sigma_2}(1,2\to 3,4)]\right]$$

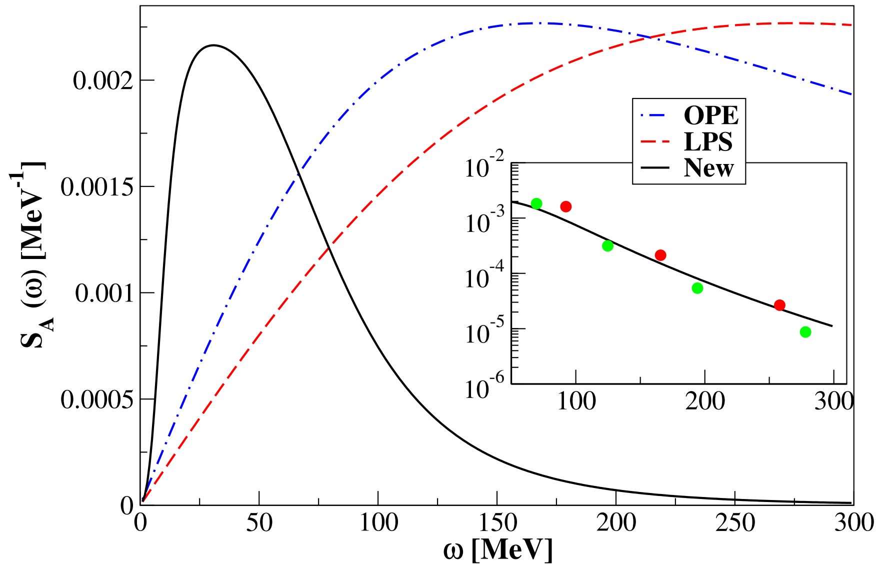
tensor force contributes

1. From Landau Fermi liquid theory
   Lykasov, Pethick, Schwenk, '08
2. Excitation above Fermi sea
3. Short range interaction – suppressing response, and reducing it to 2-particle Response.



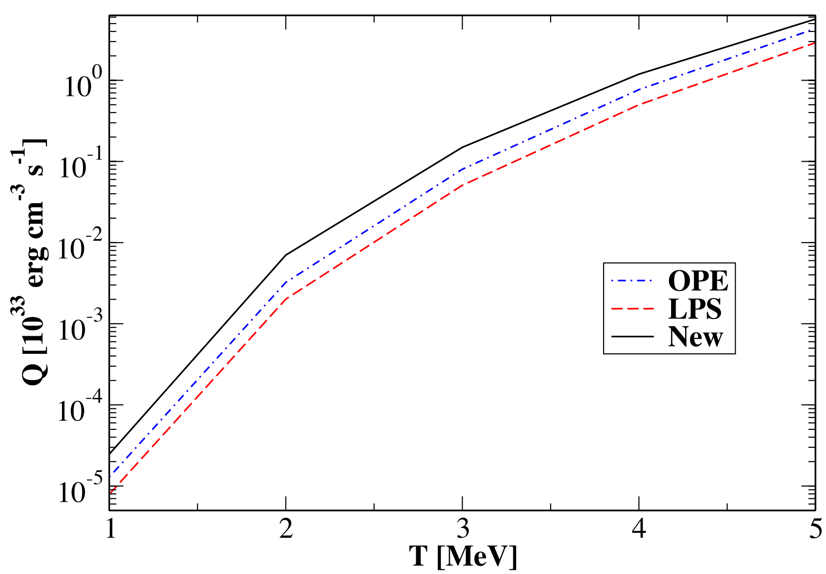
Neutrino bremsstrahlung

- T=0 spin response function is constrained by sum rule results.
- OPE, LPS based on Landau Fermi liquid theory – valid at very low energy.
- high energy behavior given by two neutrons in a box - consistent with fitted response function to the sum rules.



- valid at low T, q
- neutrino bremsstrahlung rate is even larger than OPE result.

$$Q = \frac{C_A^2 G_F^2 n}{20\pi^3}\int_0^\infty d\omega\omega^6 e^{-\omega/T} S_A(\omega)$$



Shen, Gandolfi, Reddy, Carlson(2011)