

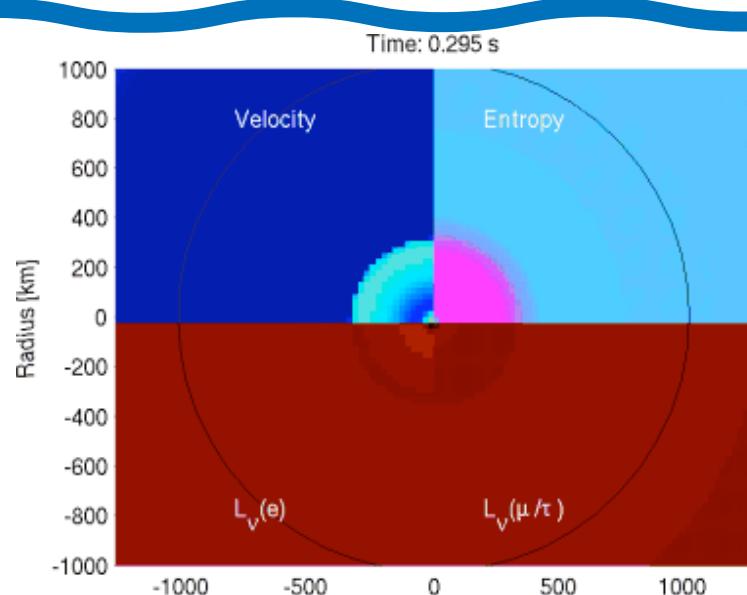
# Microphysics of Supernova Core

M. Liebendörfer  
University of Basel

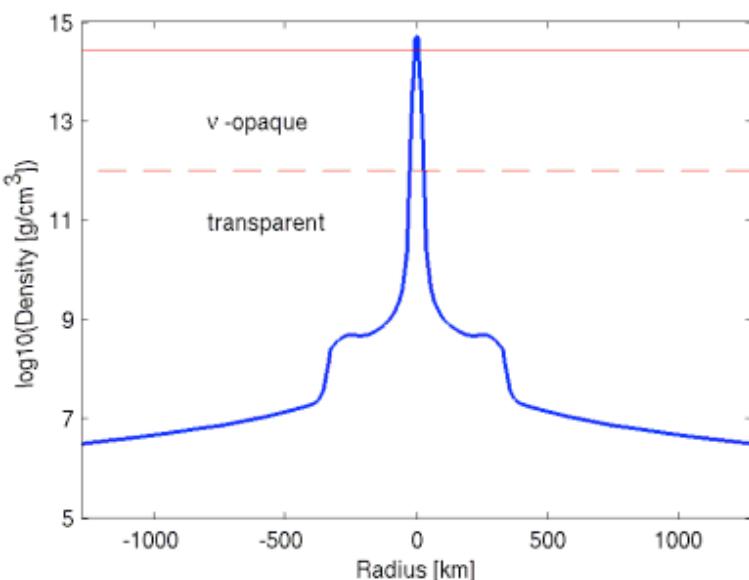
- Matter and neutrinos in the collapse phase
- Matter and neutrinos in the postbounce phase
- New 3D models of 11 Ms, 15 Ms and 40 Ms progenitor

R. Cabezon  
M. Hempel  
R. Käppeli  
A. Perego  
F.-K. Thielemann  
N. Vasset  
S. C. Whitehouse

# The Explosion Mechanism(s)



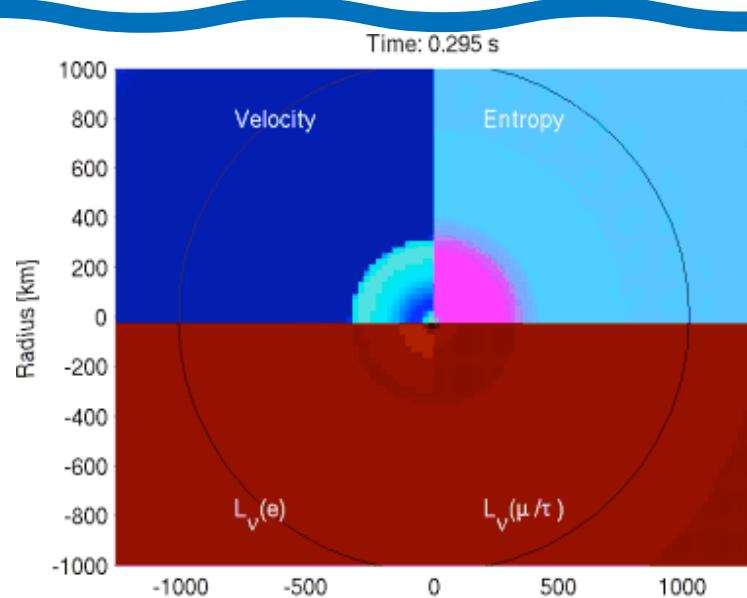
Keep in mind:  
The explosion is  
only a surface  
effect on the  
protoneutron  
star!



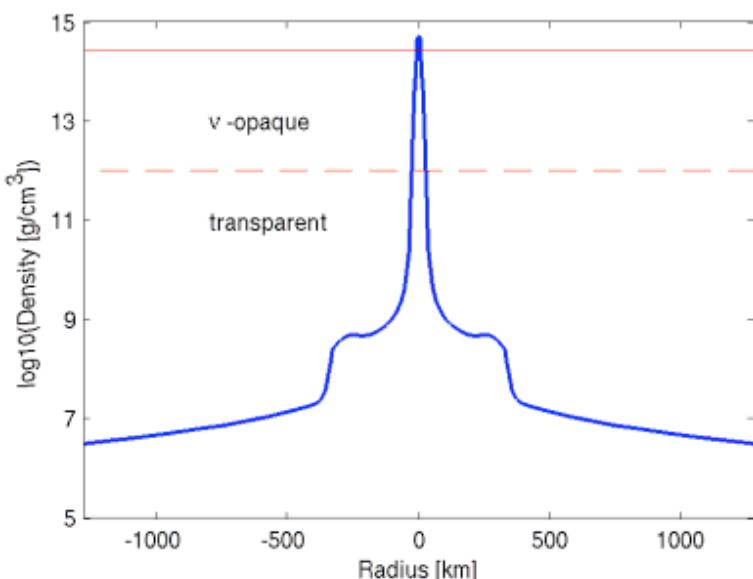
Different  
supernova  
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- prompt hydrodynamic
- delayed,  $\nu$ -driven
- magneto-rotational
- acoustic
- magnetoviscous/sonic
- phase transition in NS

# The Explosion Mechanism(s)



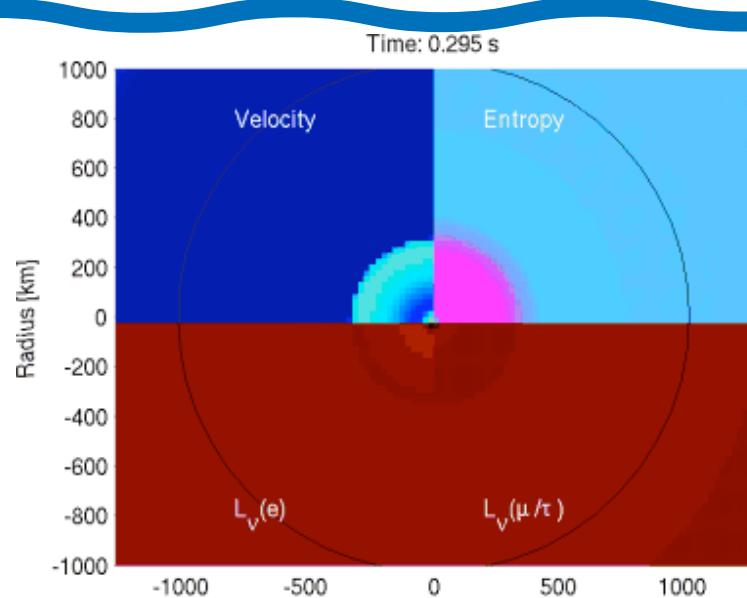
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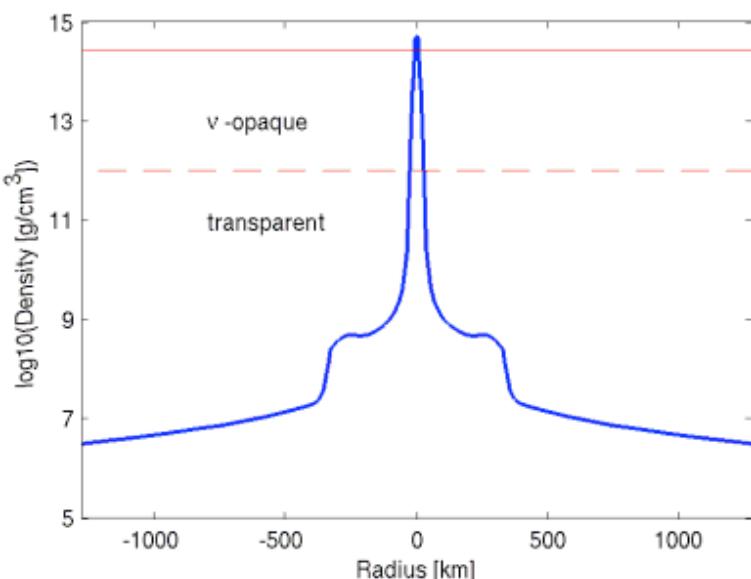
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- prompt hydrodynamic  
 (e.g. Baron et al. 1985)
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 (Colgate 1966, Bond 1979, ...  
 Marek & Janka 2009)
- magneto-rotational  
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- phase transition in NS  
 (Migdal et al. 1971, ...  
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# The Explosion Mechanism(s)



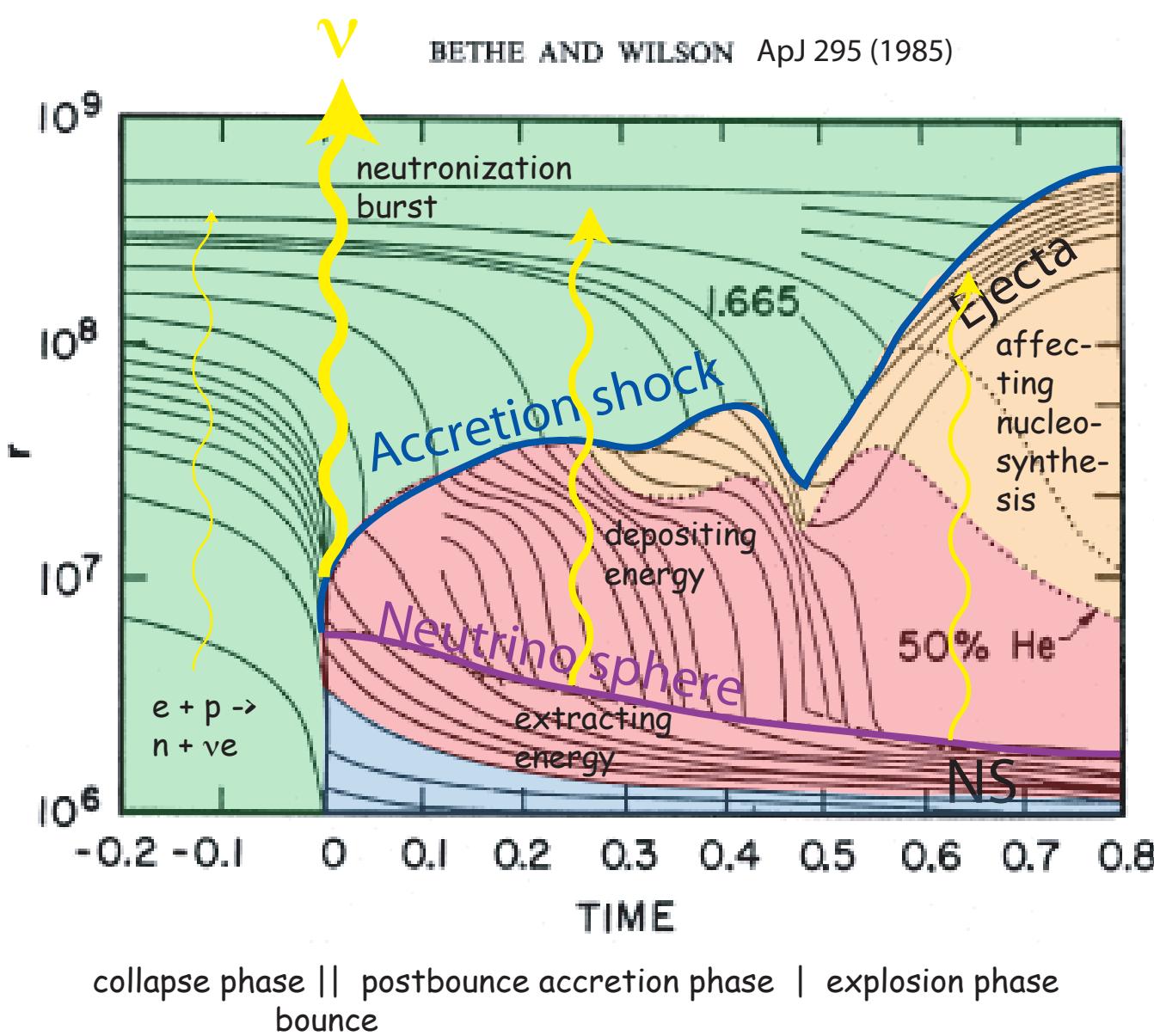
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# Matter conditions



Ensemble  
of nuclei

Cool bulk  
nuclear matter

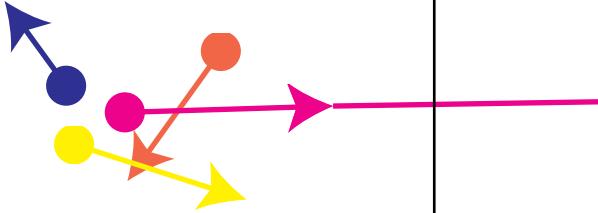
Hot dissociated  
matter

Freeze-out  
of nuclei

# Neutrino-matter interactions

Bruenn (1985)  
Raffelt (2001)

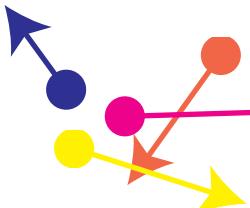


Description:	Number Sphere	Energy Sphere	Transport Sphere
Emission & Absorption			
			
Can be Pauli-blocked in diffusive regime			
Production/Annihilation			
Electron or Positron capture			
$\nu_e + n \rightleftharpoons e^- + p$			
$\bar{\nu}_e + p \rightleftharpoons e^+ + n$			
$\nu_e + (A, Z) \rightleftharpoons e^- + (A, Z + 1)$			
NN bremsstrahlung (Thompson et al. 2002)			
$\nu e$ pair $\rightarrow \nu \mu$ pair (Buras et al. 2003)			
High matter density			Low matter density

# Neutrino-matter interactions

Bruenn (1985)  
Raffelt (2001)

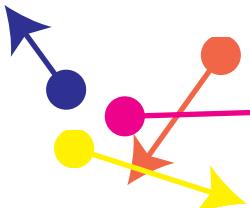


Description:	Number Sphere	Energy Sphere	Transport Sphere
Emission & Absorption	Inelastic Scattering		
			
Can be Pauli-blocked in diffusive regime	Probability enhanced by isoenergetic scattering	Thermalisation	
Production/Annihilation			
Electron or Positron capture			
$\nu_e + n \rightleftharpoons e^- + p$	Neutrino-electron scattering		
$\bar{\nu}_e + p \rightleftharpoons e^+ + n$	$\nu + e \rightleftharpoons \nu + e$		
$\nu_e + (A, Z) \rightleftharpoons e^- + (A, Z + 1)$	Inelastic neutrino-nucleus scattering		
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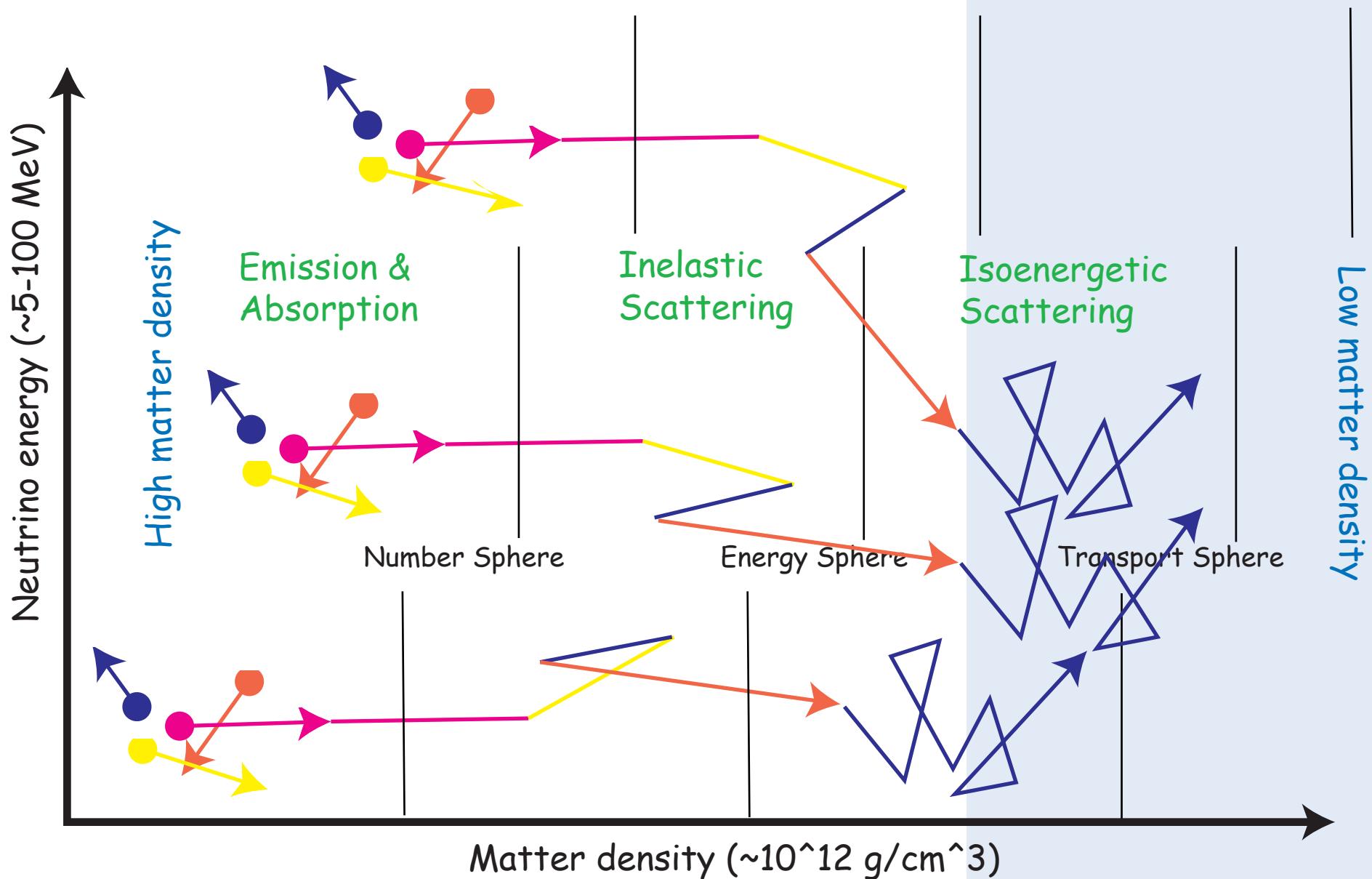
# Neutrino-matter interactions

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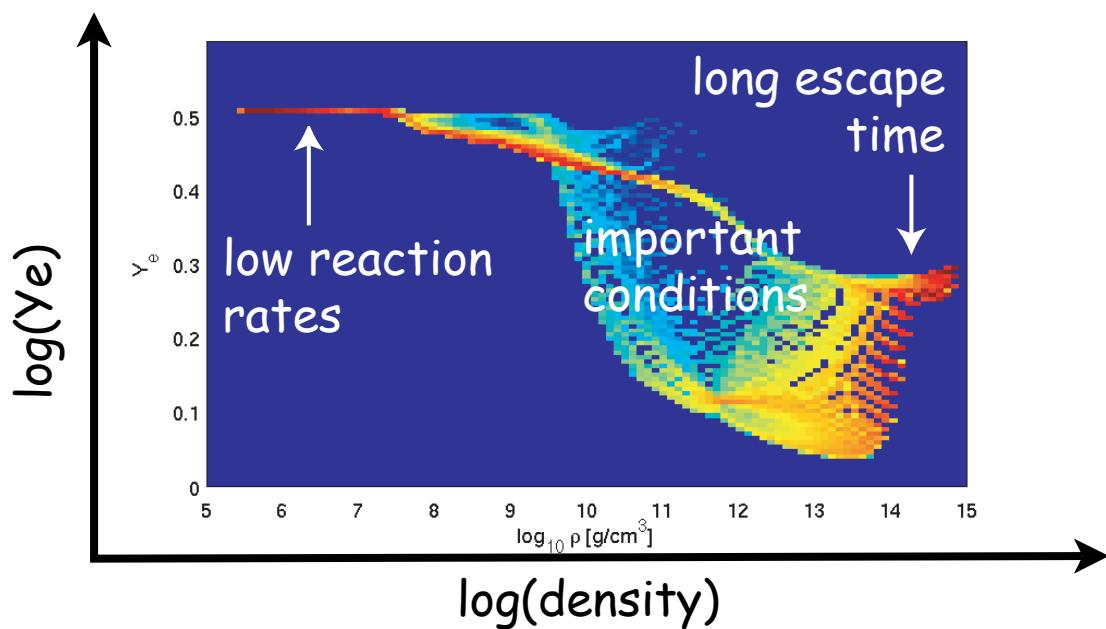
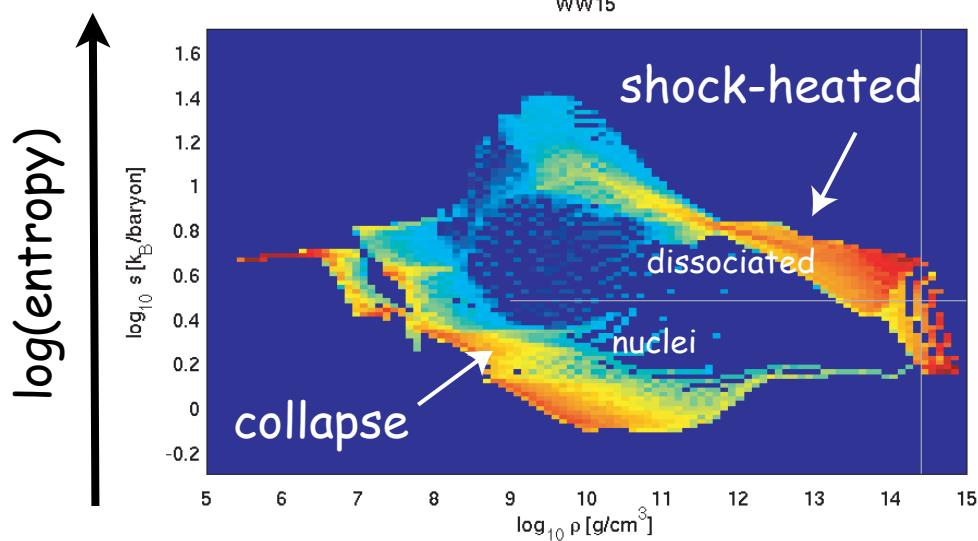


Description:	Number Sphere	Energy Sphere	Transport Sphere
Emission & Absorption	Inelastic Scattering	Isoenergetic Scattering	
 Can be Pauli-blocked in diffusive regime	Probability enhanced by isoenergetic scattering Thermalisation	Strong energy dependence, $\sim E^2$ Scattering	Low matter density Streaming
Production/Annihilation Electron or Positron capture $\nu_e + n \rightleftharpoons e^- + p$ $\bar{\nu}_e + p \rightleftharpoons e^+ + n$ $\nu_e + (A, Z) \rightleftharpoons e^- + (A, Z + 1)$ NN bremsstrahlung (Thompson et al. 2002) $\nu e$ pair $\rightarrow \nu \mu$ pair (Buras et al. 2003)	Neutrino-electron scattering $\nu + e \rightleftharpoons \nu + e$ Inelastic neutrino-nucleus scattering	Elastic coherent scattering of neutrinos on nuclei $\nu + (A, Z) \rightleftharpoons \nu + (A, Z)$ Neutrino-nucleon scattering $\nu + N \rightleftharpoons \nu + N$	

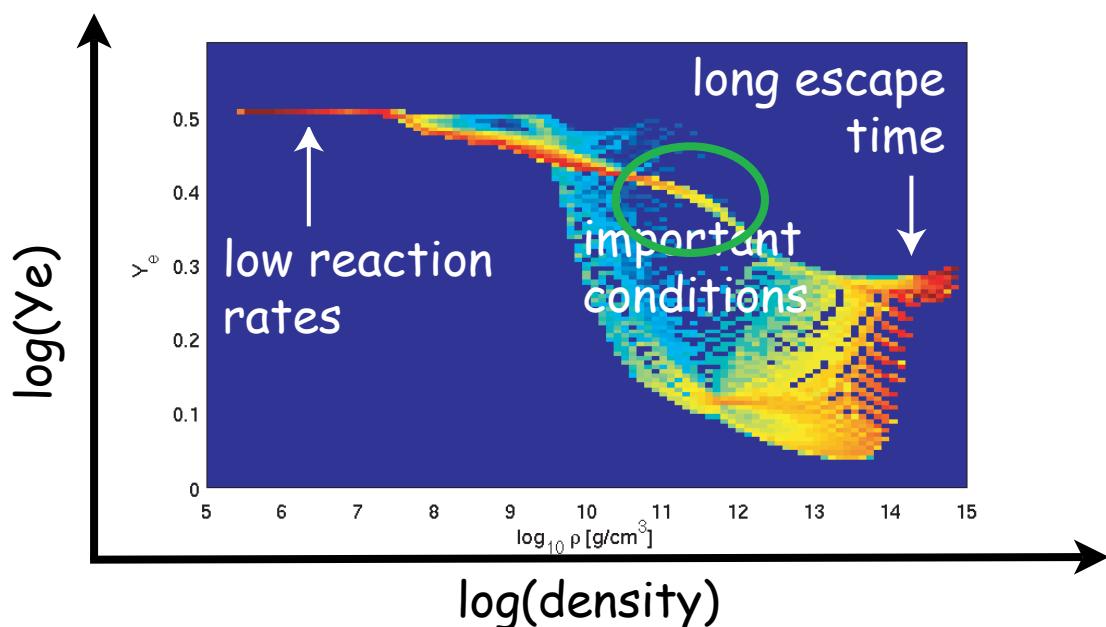
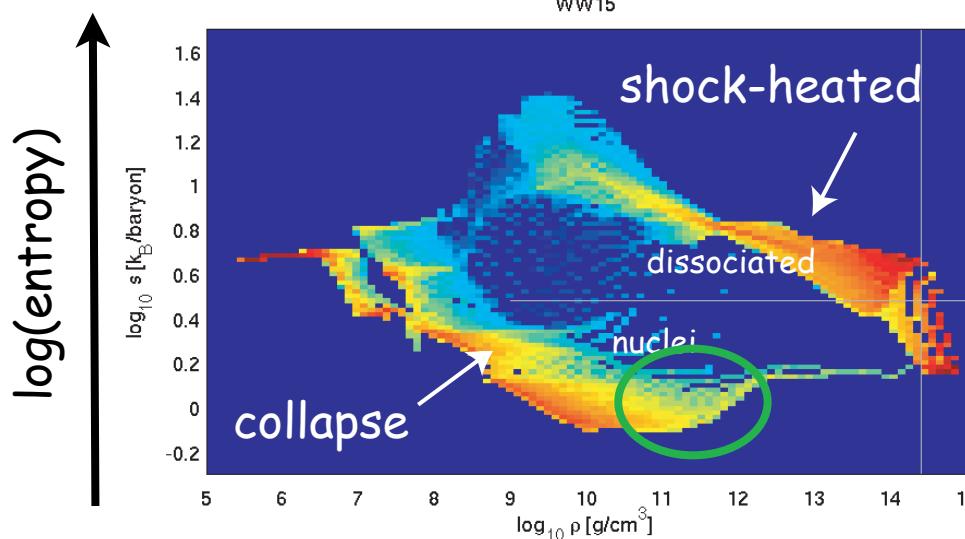
# Energy-dependent neutrino transport



# Relevant $\nu$ -matter interactions



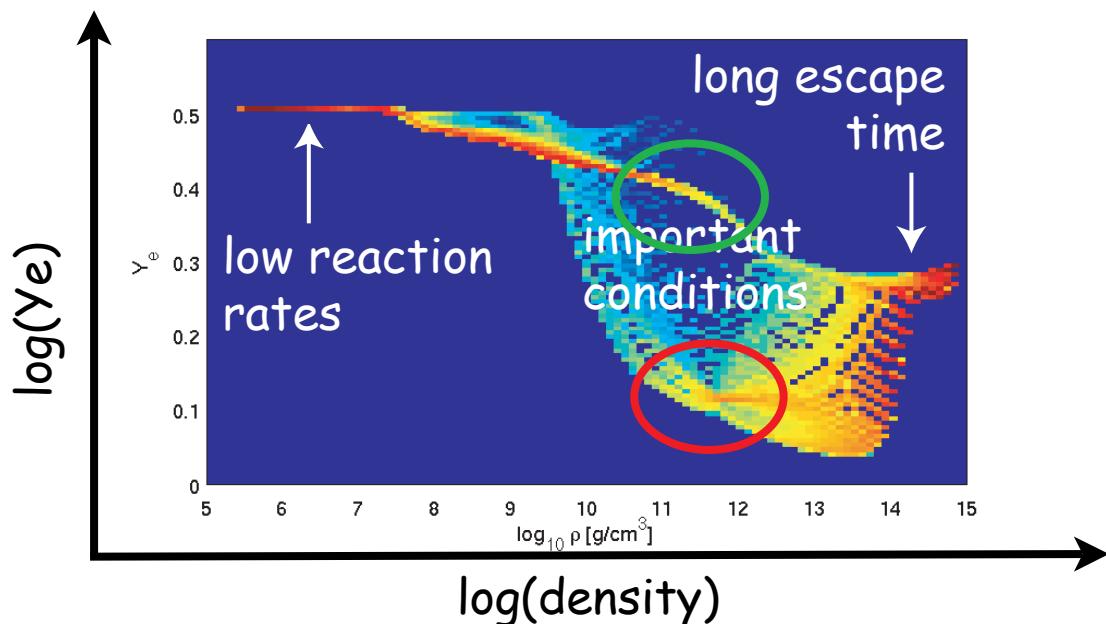
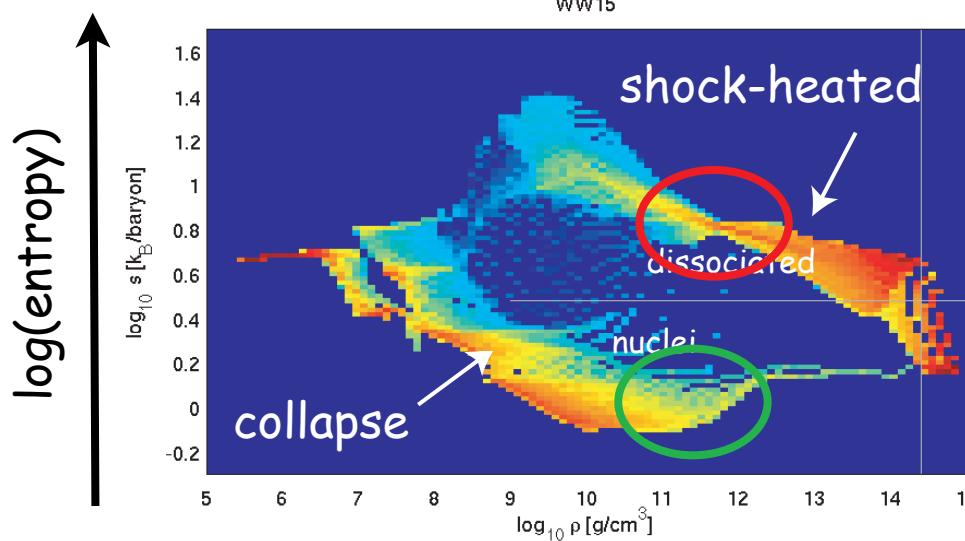
# Relevant $\nu$ -matter interactions



The conditions around the neutrino spheres are marked in

green ... collapse

# Relevant $\nu$ -matter interactions

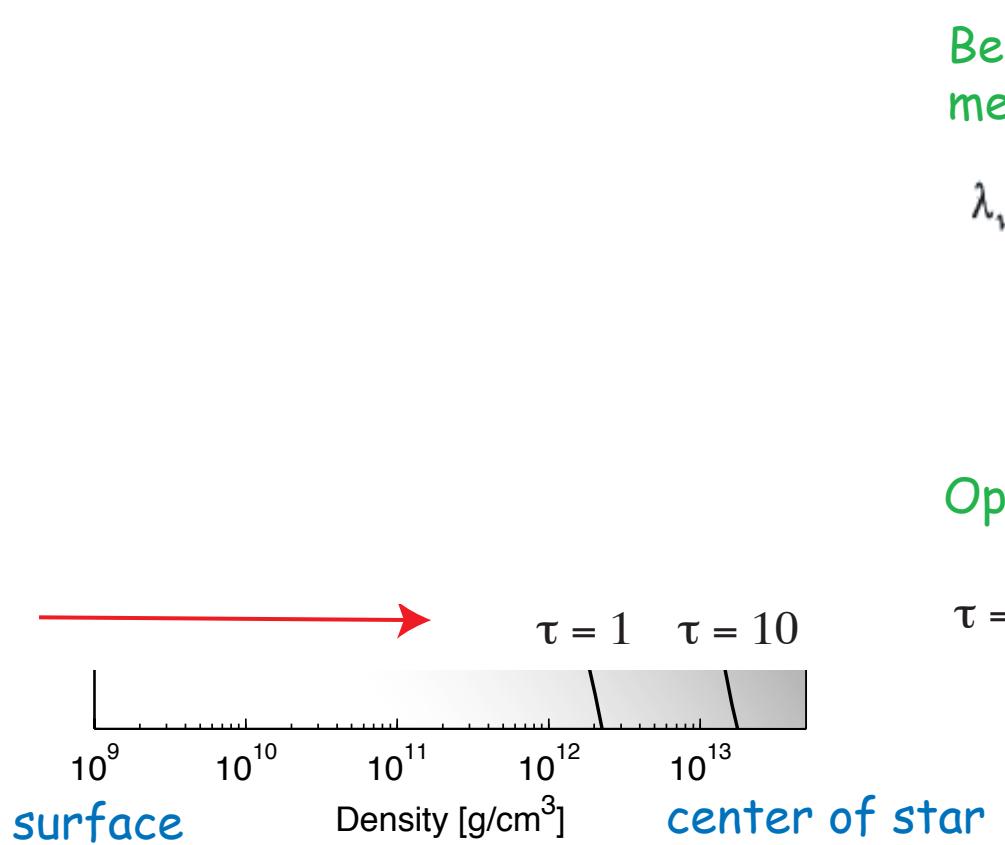


The conditions around the neutrino spheres are marked in

green ... collapse

red ... postbounce

# Physics for the collapse phase



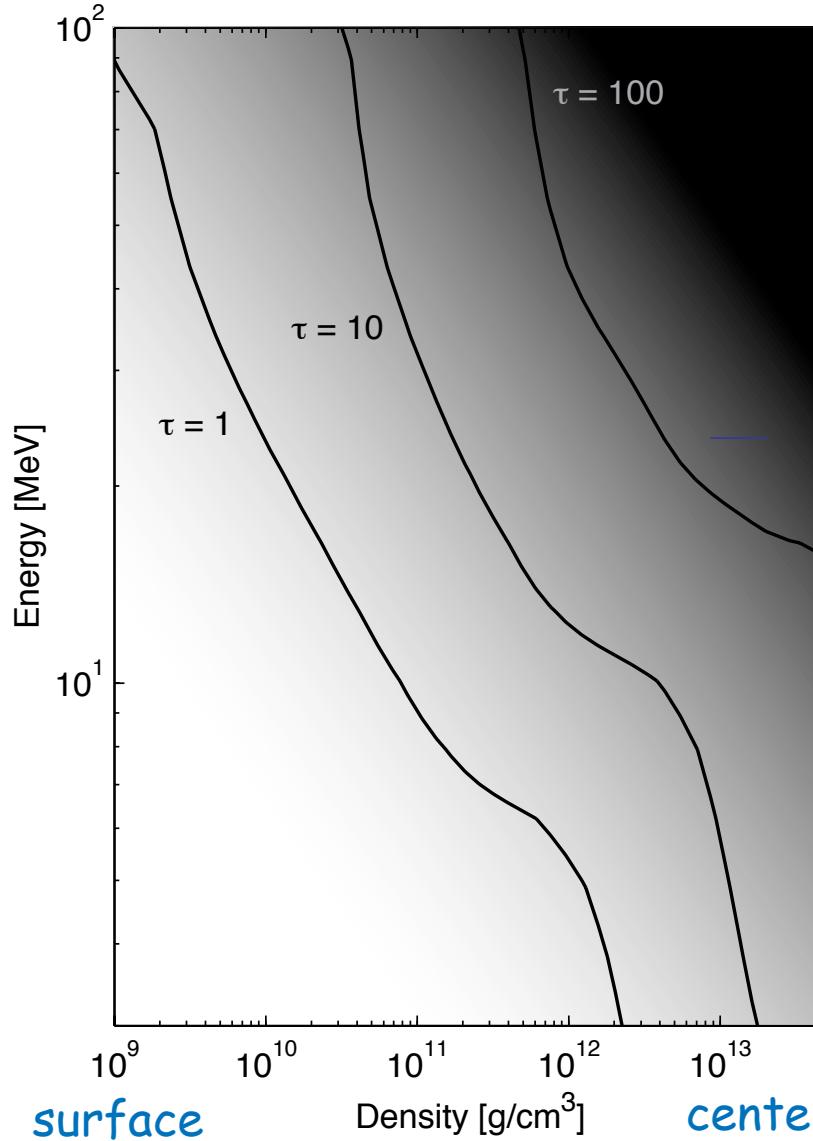
Bethe (1990)  
mean free path:

$$\lambda_\nu = 1.0 \times 10^8 \rho_{12}^{-1} [(N^2/6A)X_h + X_n]^{-1} \varepsilon_\nu^{-2} \text{ cm}$$

## Optical depth:

$$\tau = \int dr/\lambda$$

# Deleptonization



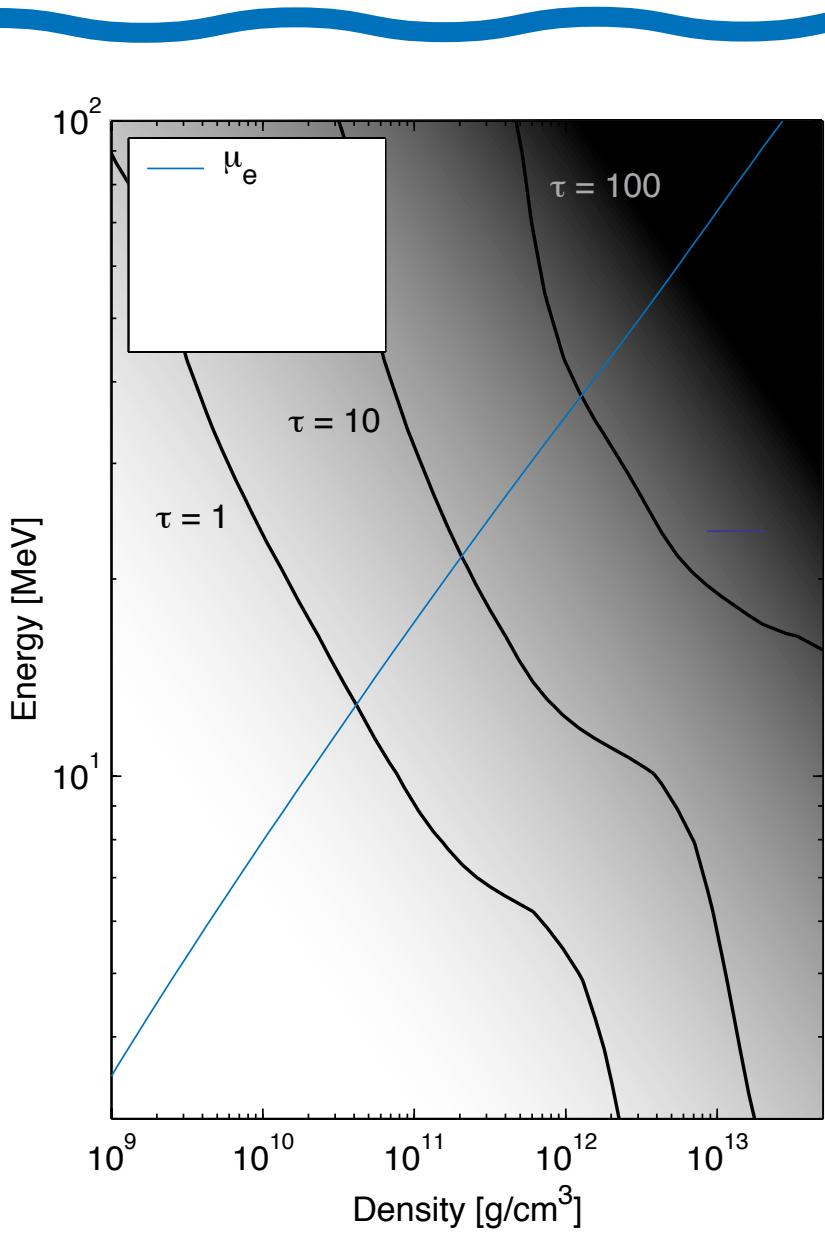
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Optical depth:

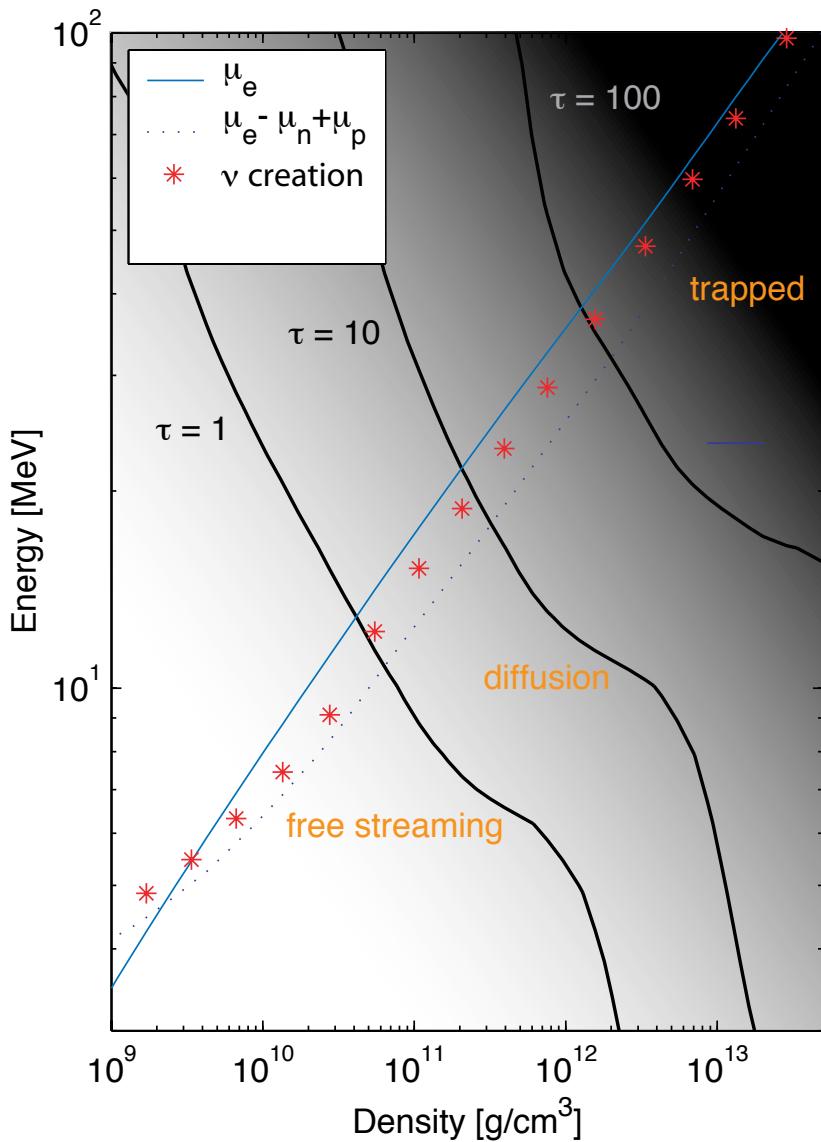
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# Deleptonization



(Martinez-Pinedo, Liebendoerfer, Frekers, 2006)

# Deleptonization

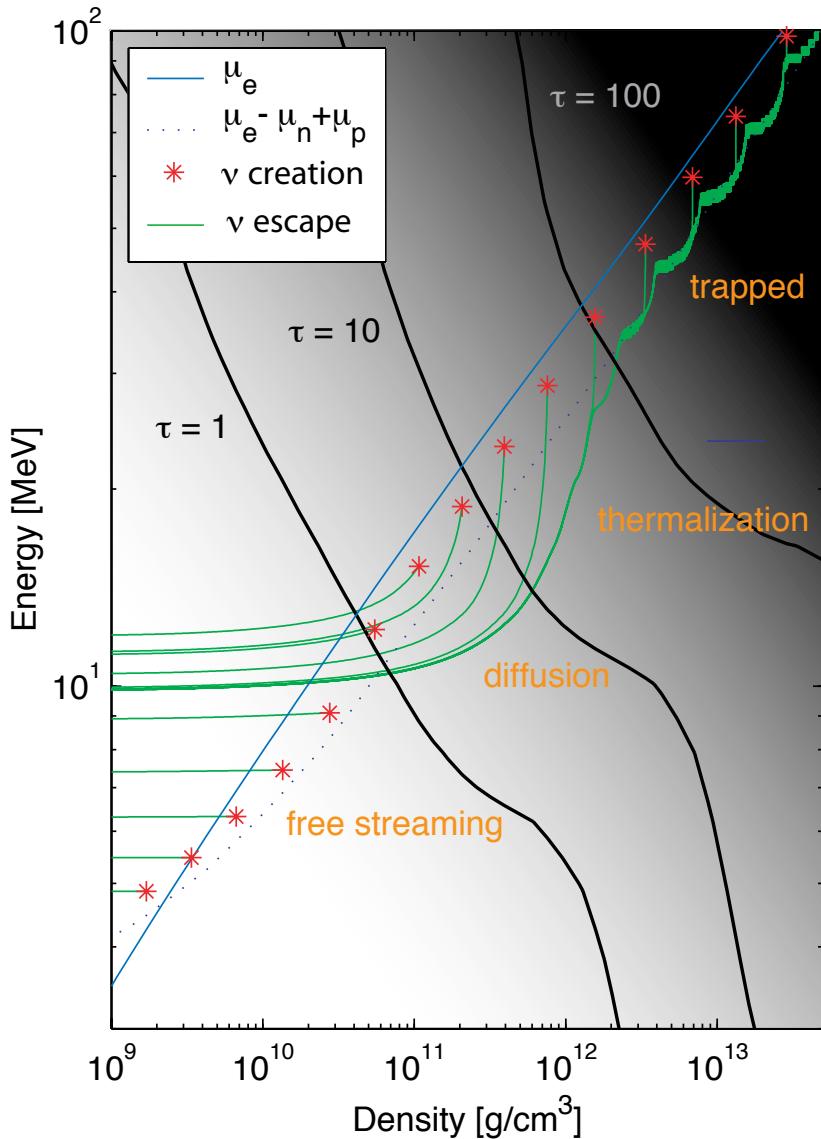


$$\mu_\nu \sim \mu_e - \mu_n + \mu_p$$

$$\frac{\Delta s}{\Delta t} = -\frac{\Delta Y_e}{\Delta t} \frac{\mu_e - \mu_n + \mu_p - E_\nu^{\text{esc}}}{T}$$

(Martinez-Pinedo, Liebendoerfer, Frekers, 2006)

# Deleptonization



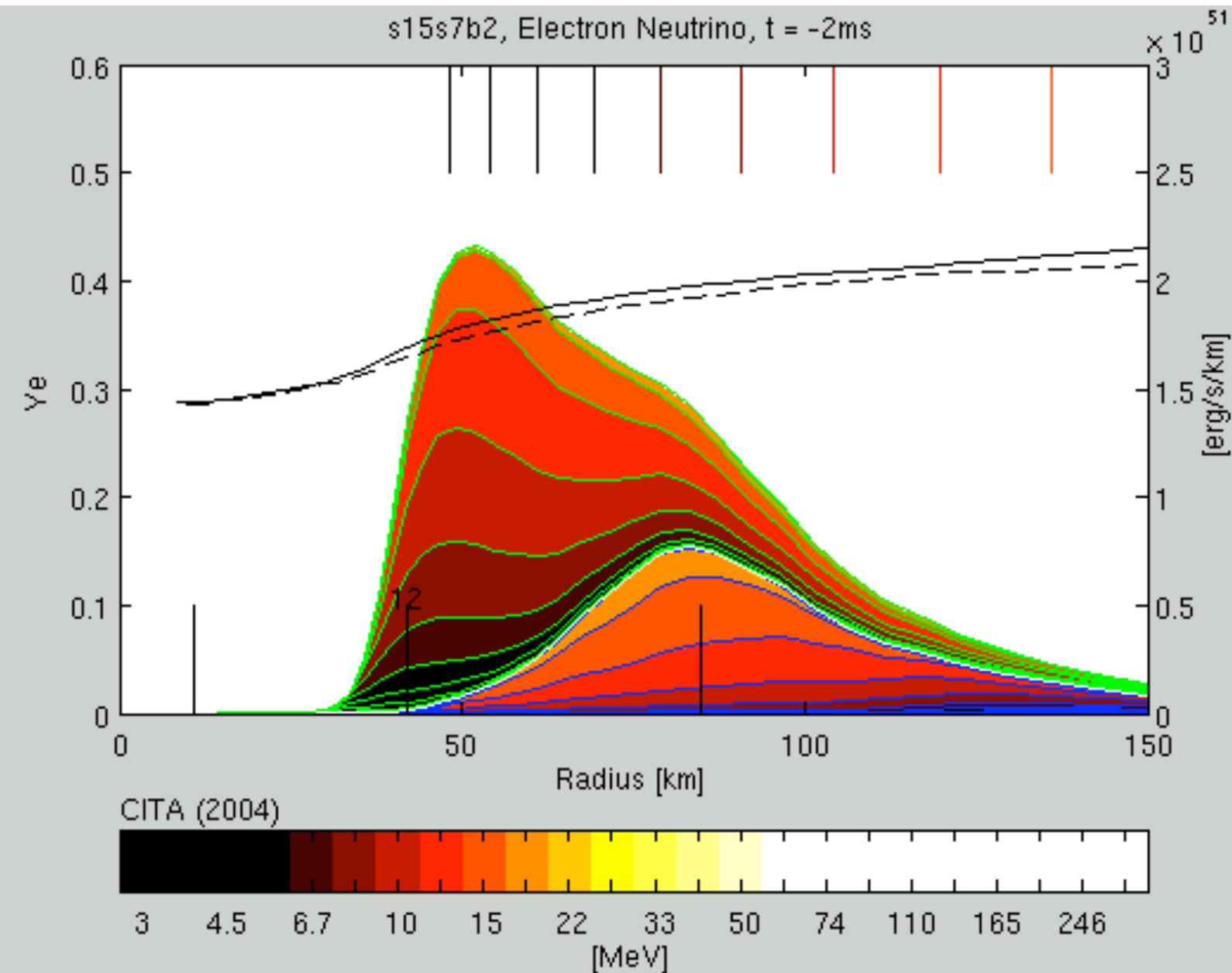
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$\nu$ 's escape directly:  
 -> entropy decrease

$\nu$ 's thermalise:  
 -> entropy increase

# Histogram of electron neutrino emission

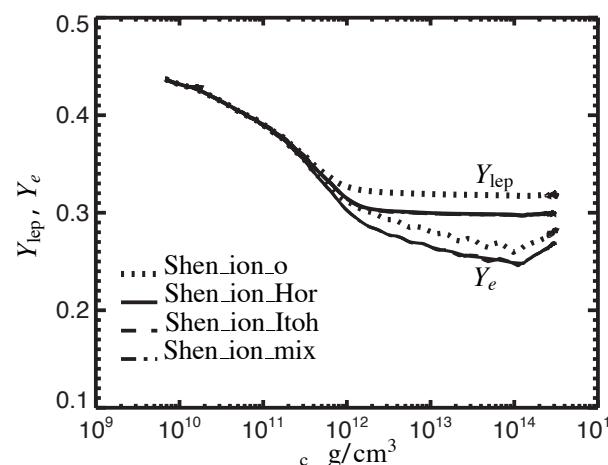
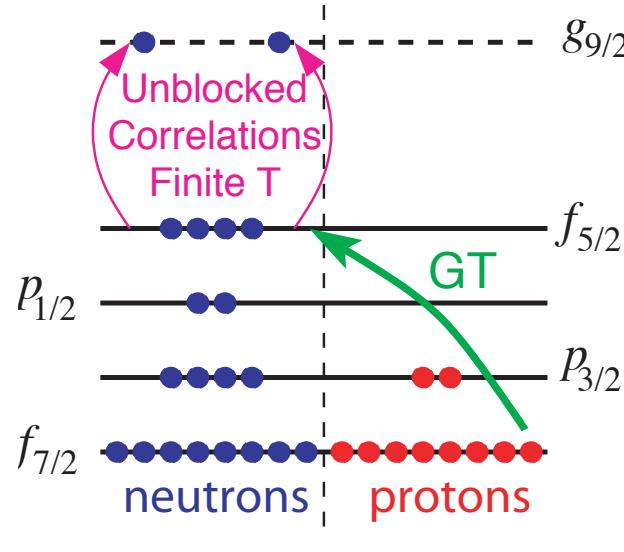


# Physics for the collapse phase

Electron capture rates  
 - ensemble of nuclei  
 with individual rates

Thermalisation  
 - neutrino-electron  
 scattering, inelastic  
 scattering with nuclei

Opacity  
 - coherent scattering  
 - ion-ion correlations  
 - clustering in phase  
 transition  
 - unresolved: how to  
 scatter on ensemble of  
 nuclei?



Langake et al. 2003, Hix et al.  
 2003, Martinez-Pinedo,  
 Liebendorfer, Frekers 2006

Myra & Bludman, Bruenn 1989,  
 Langanke et al. 2008

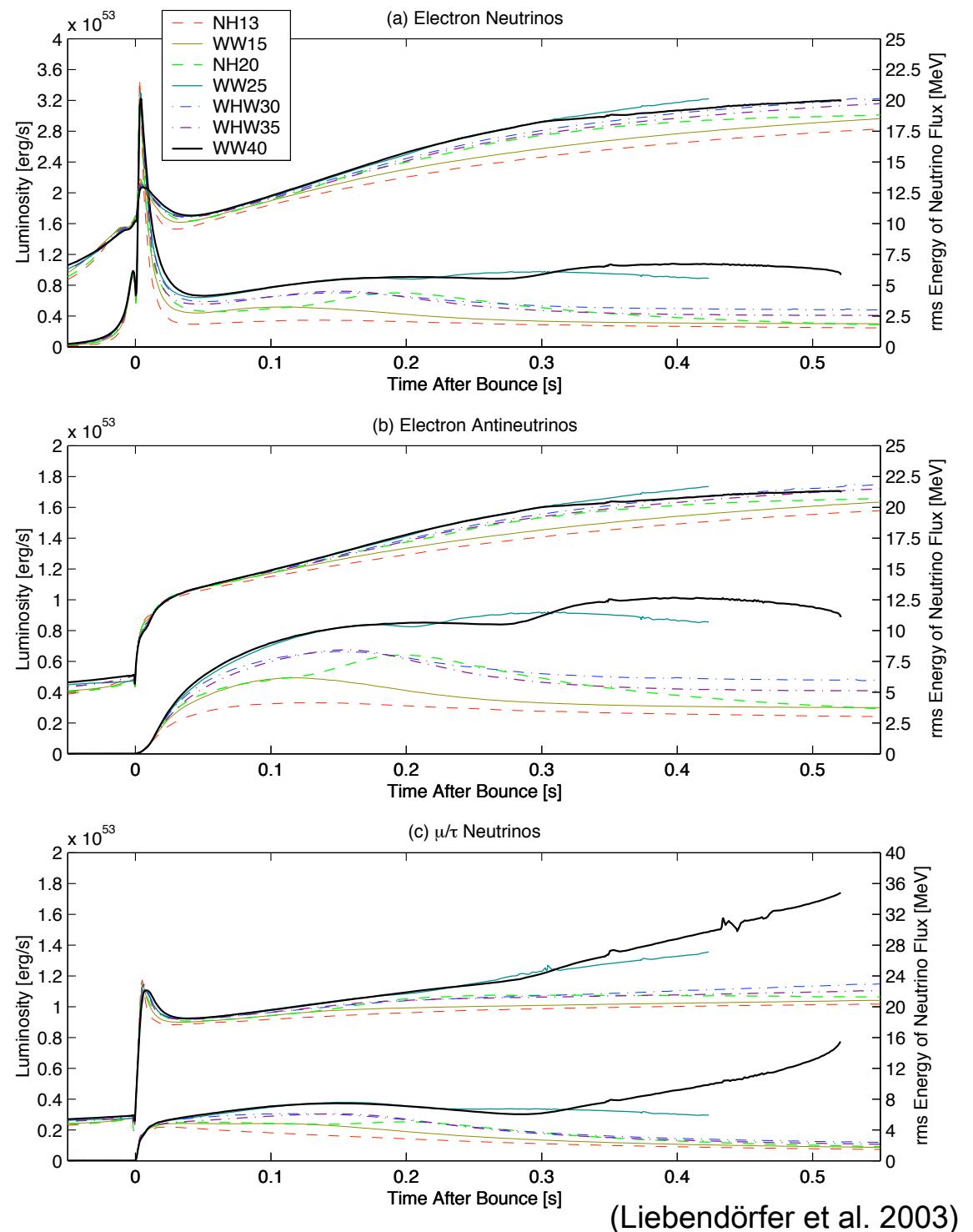
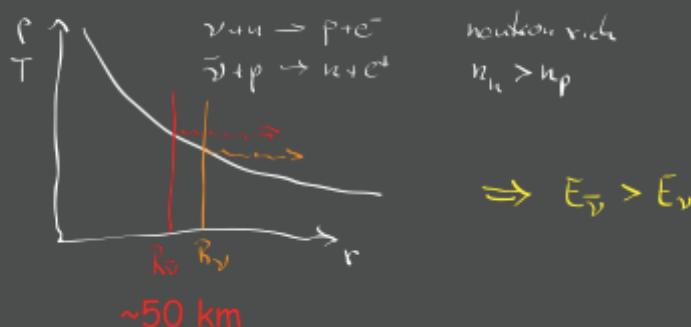
Itoh 1975, Horowitz 1997, Bruenn  
 & Mezzacappa 1997, Watanabe  
 2004, Horowitz et al. 2004,  
 Botvian & Mishustin 2005, Marek  
 et al. 2005

**Structure and  
 dynamics of nuclei  
 are key!**

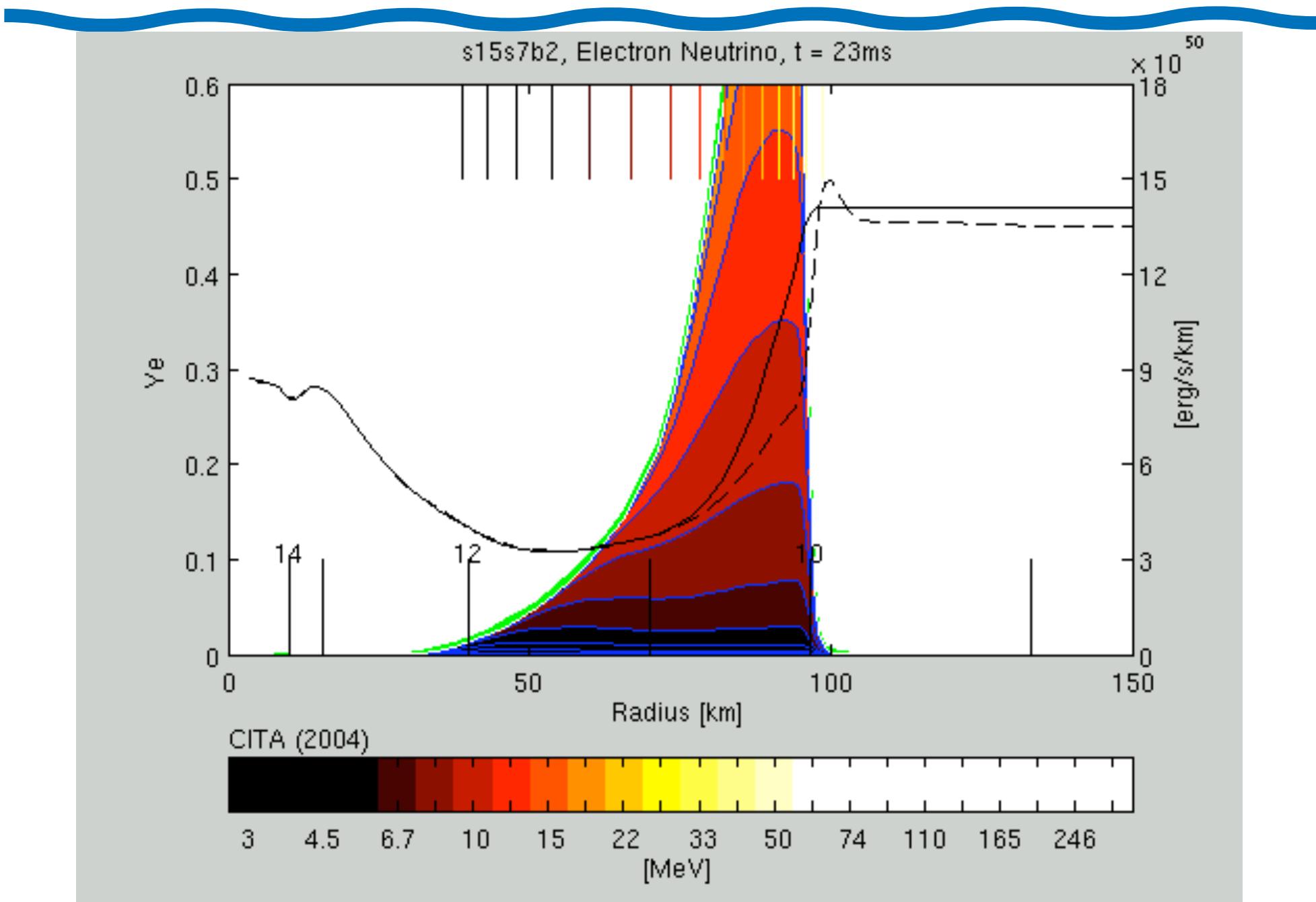
# Neutrinos from the postbounce phase

The neutrino luminosities reflect the accretion rate and the thermodynamic conditions at the neutrinospheres

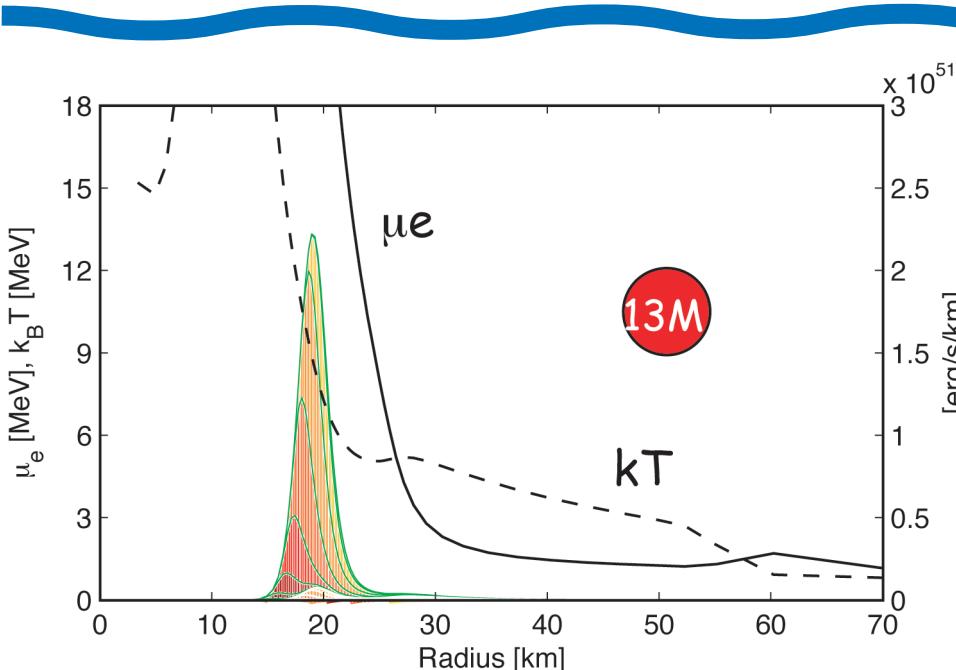
Typical energy hierarchy of neutrino energies:



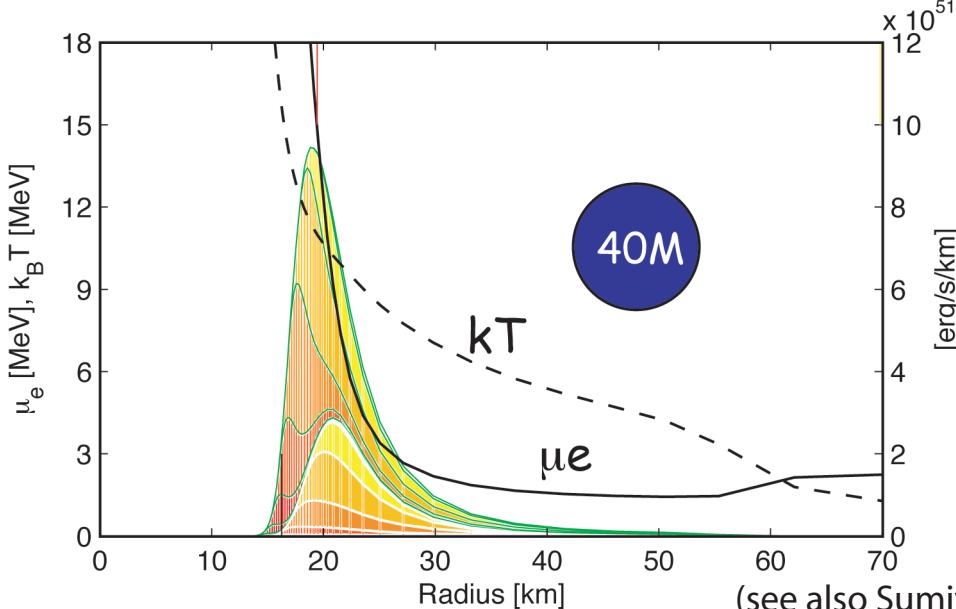
# Histogram of electron neutrino emission



# PNS evolution & $\nu(\mu/\tau)$ properties

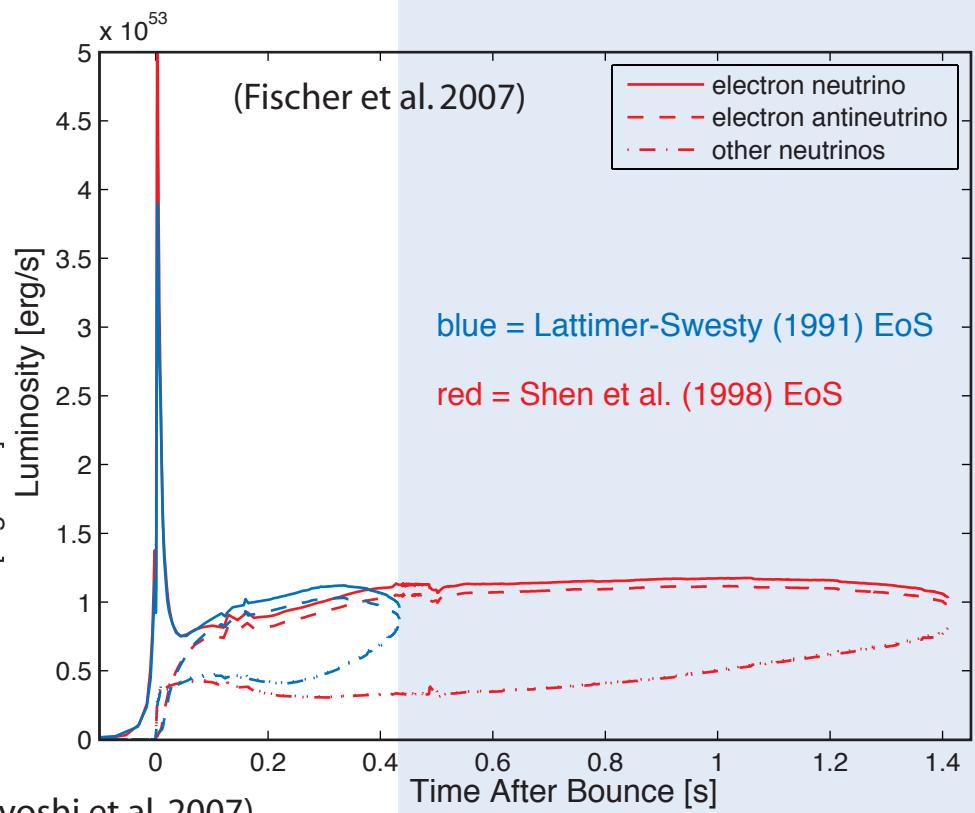


- low mass proto-neutron star (PNS)  
--> incompressible accretion



(see also Sumiyoshi et al. 2007)

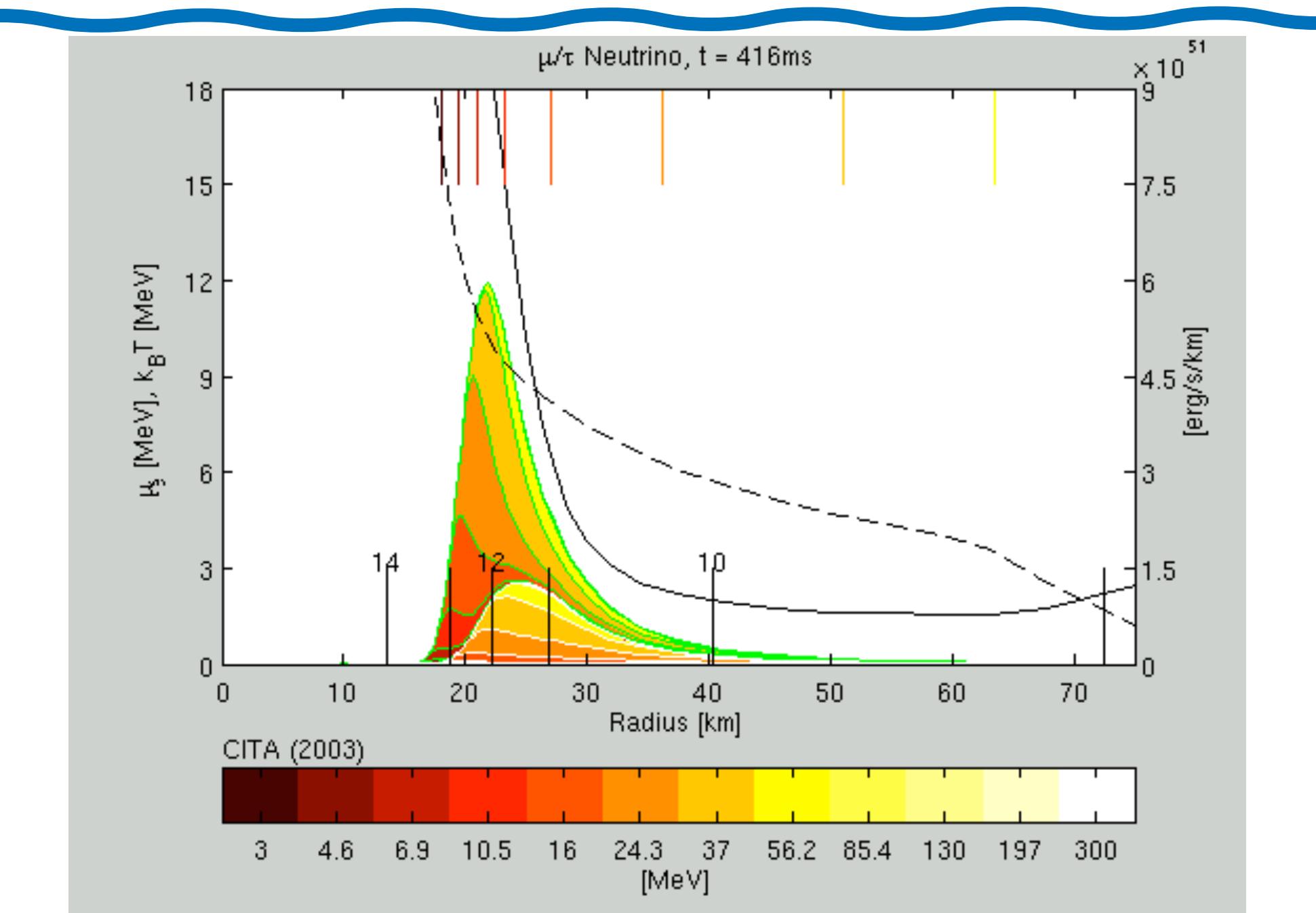
- PNS close to maximum mass  
--> hot layers pushed inward



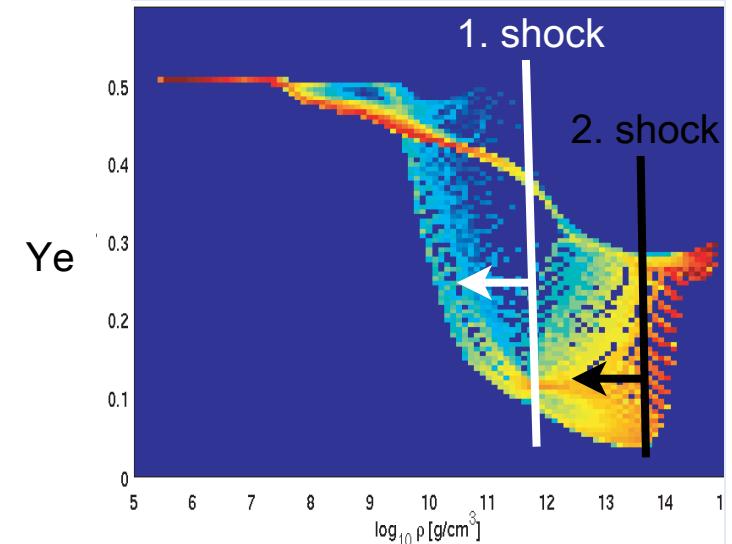
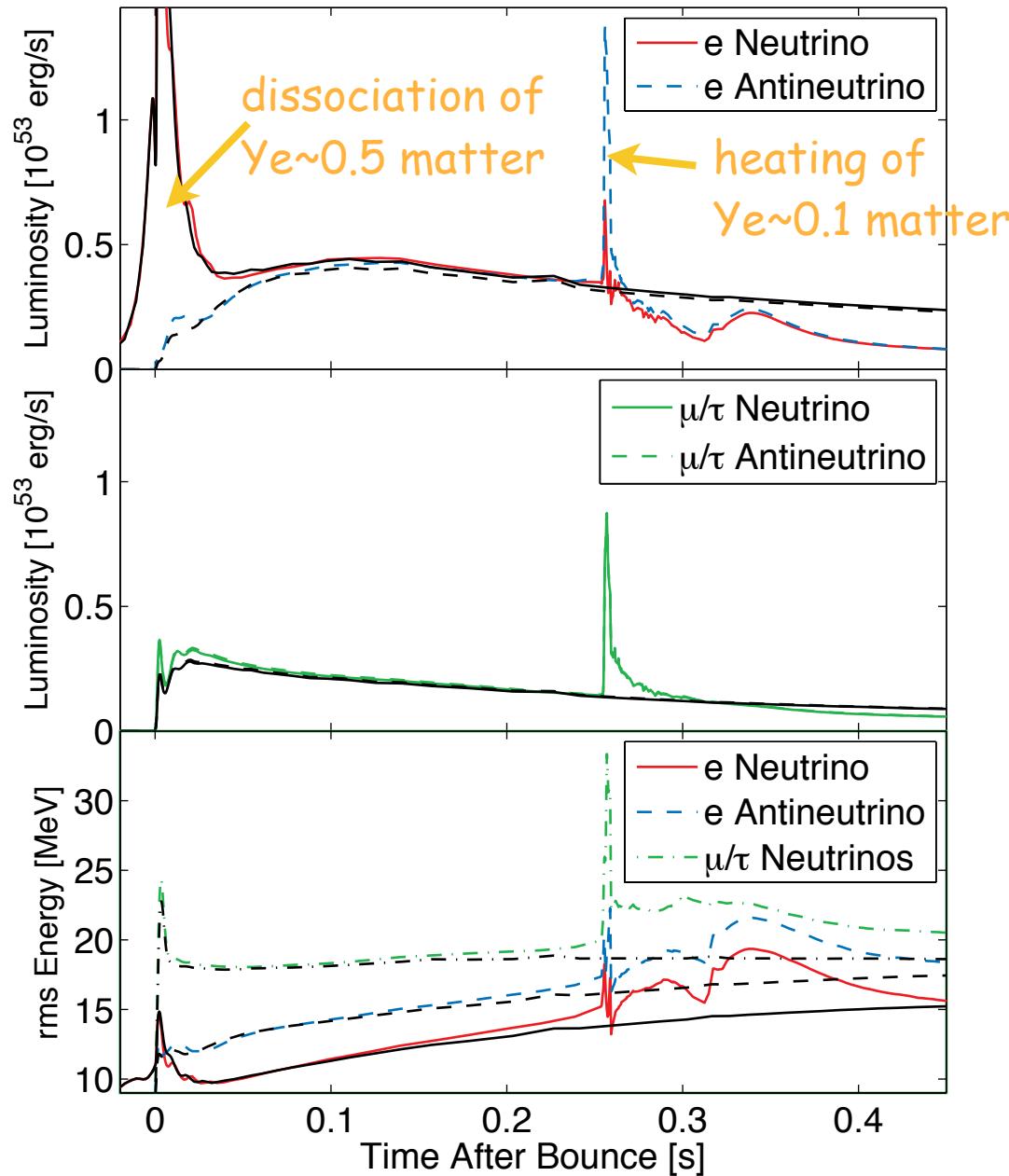
blue = Lattimer-Swesty (1991) EoS  
red = Shen et al. (1998) EoS

(Fischer et al. 2007)

# Histogram of $\mu/\tau$ neutrino emission



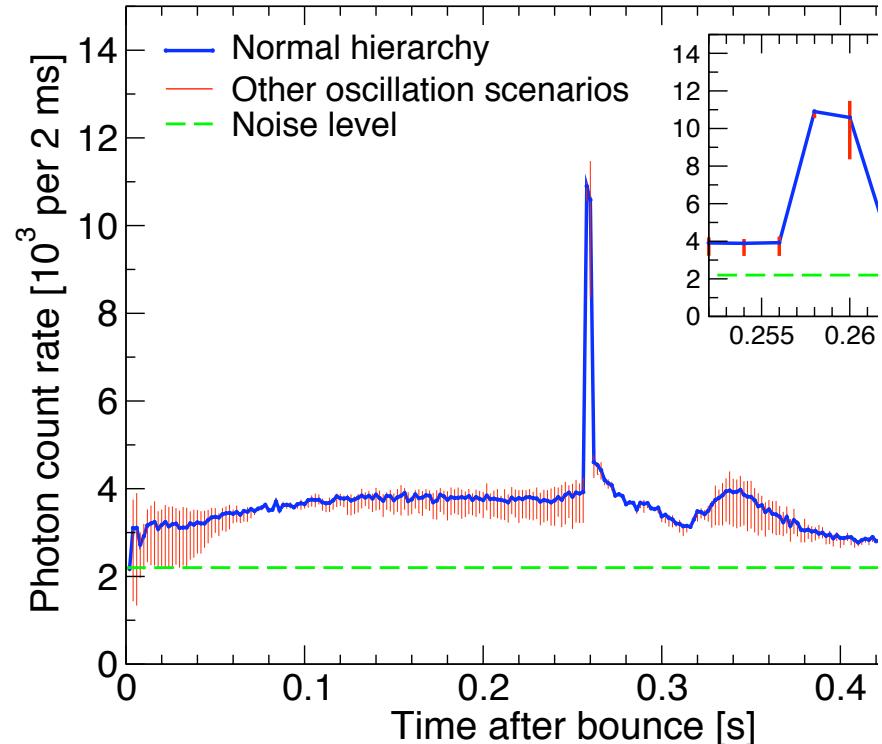
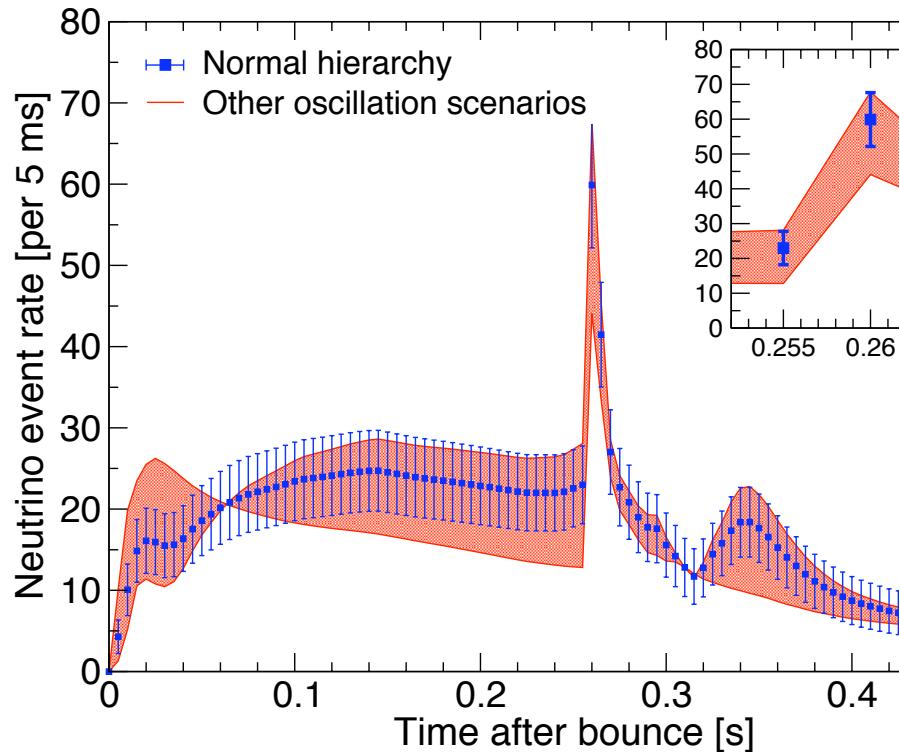
# QCD phase transition induced explosions



- Second neutrino peak in all flavours, dominated by anti- $\nu$ 's
- Step up in neutrino rms energies

(Sagert, Fischer et al., PRL 2009)

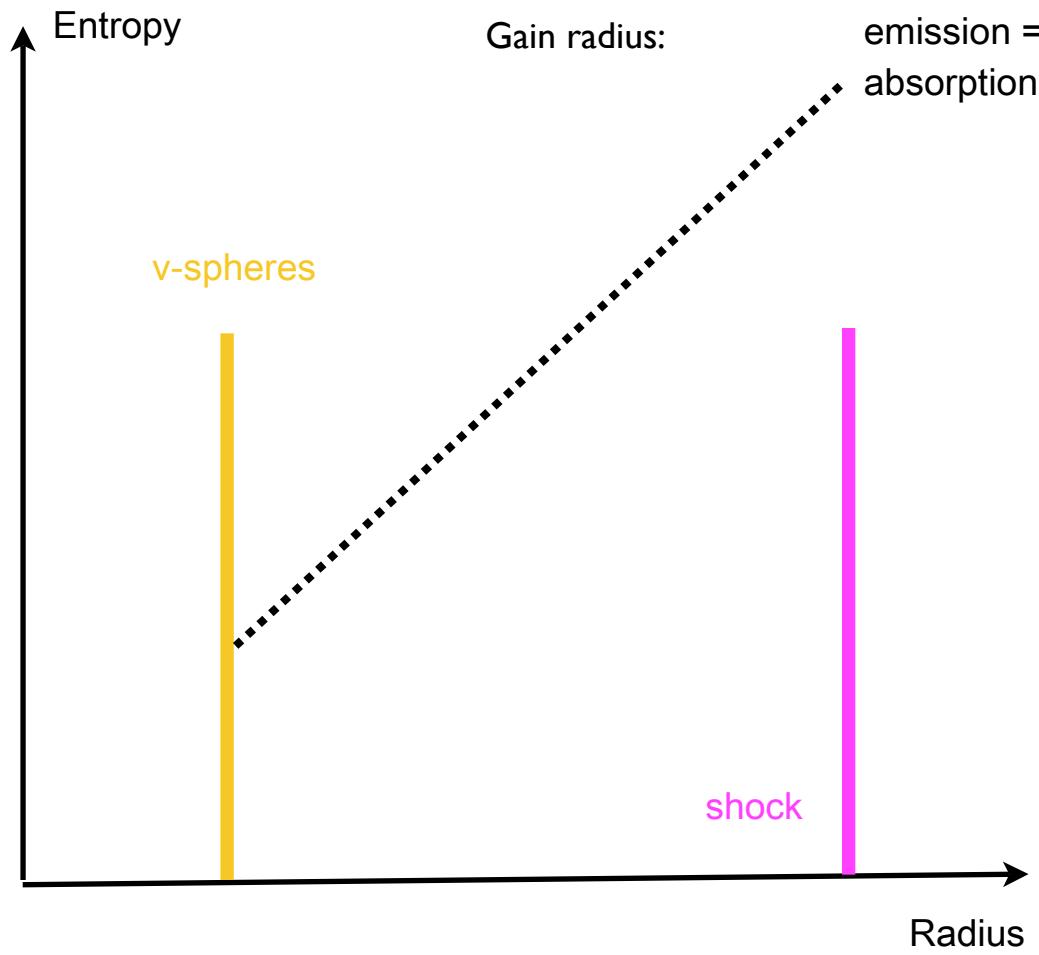
# SN as high-density physics laboratory



Neutrino signature from early QCD phase transition  
 in proto-neutron star (EOS not compatible with 1.97  
 Ms neutron star! (Demorest et al. 2010) )

(Dasgupta et al. 2010)

# Physics for the postbounce phase



There are no heavy nuclei between the neutrinosphere and the shock!

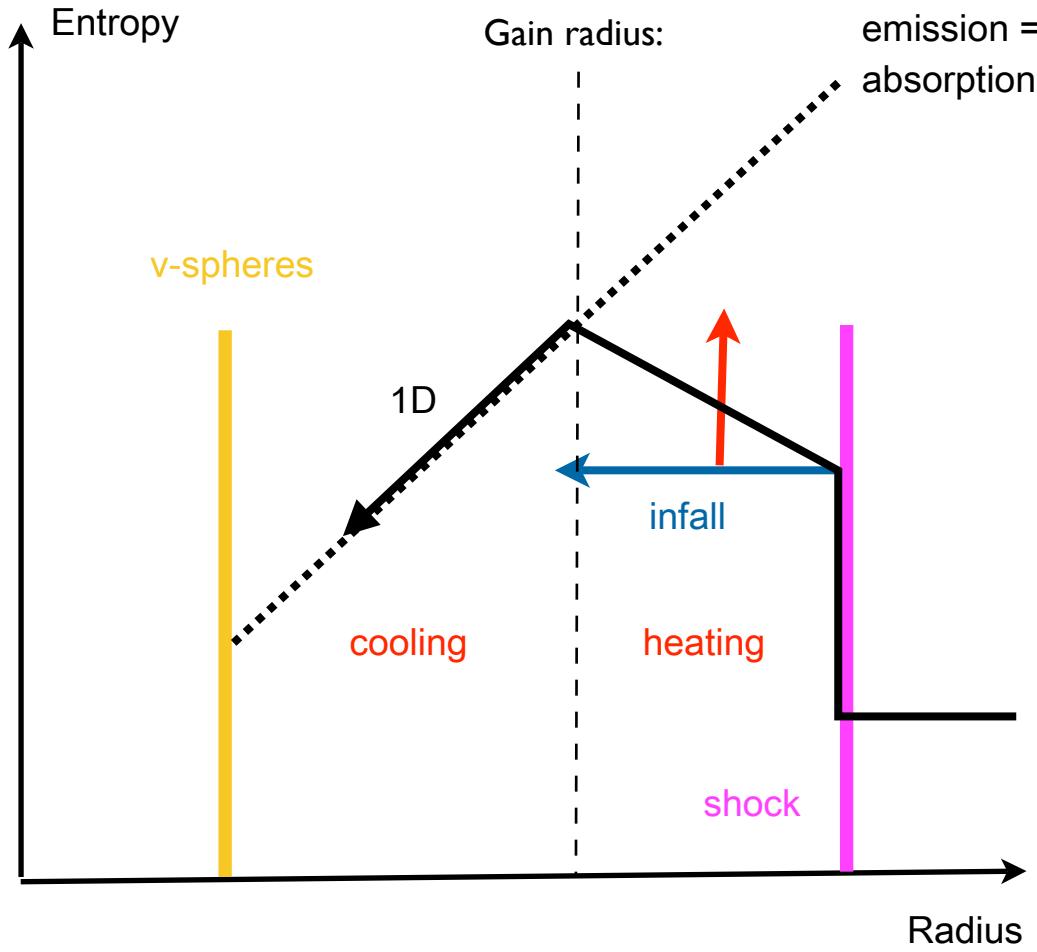
Key input:

- high-density EOS
- fluid instabilities
- neutrino transport,  
(and oscillations?)
- magnetic fields

Fluid overturn is  
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heating efficiency

(Herant et al. 1994)

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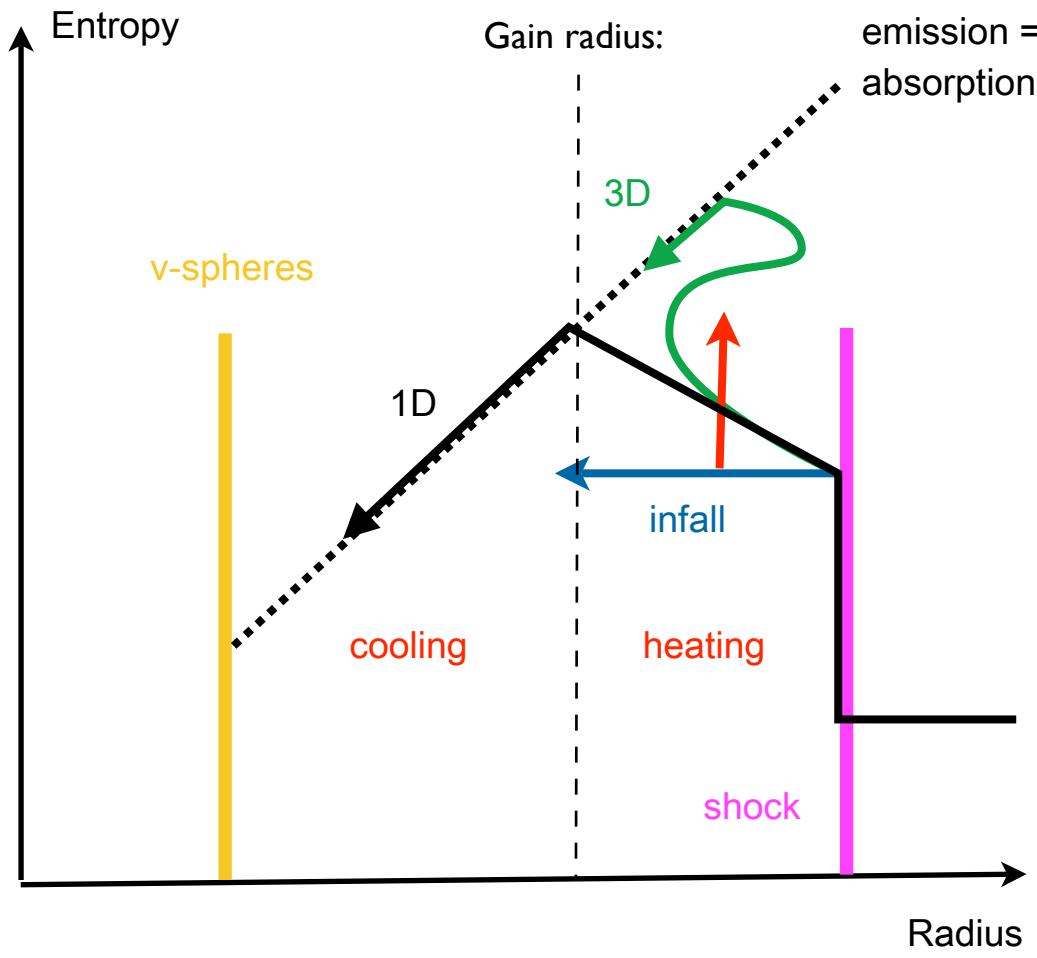
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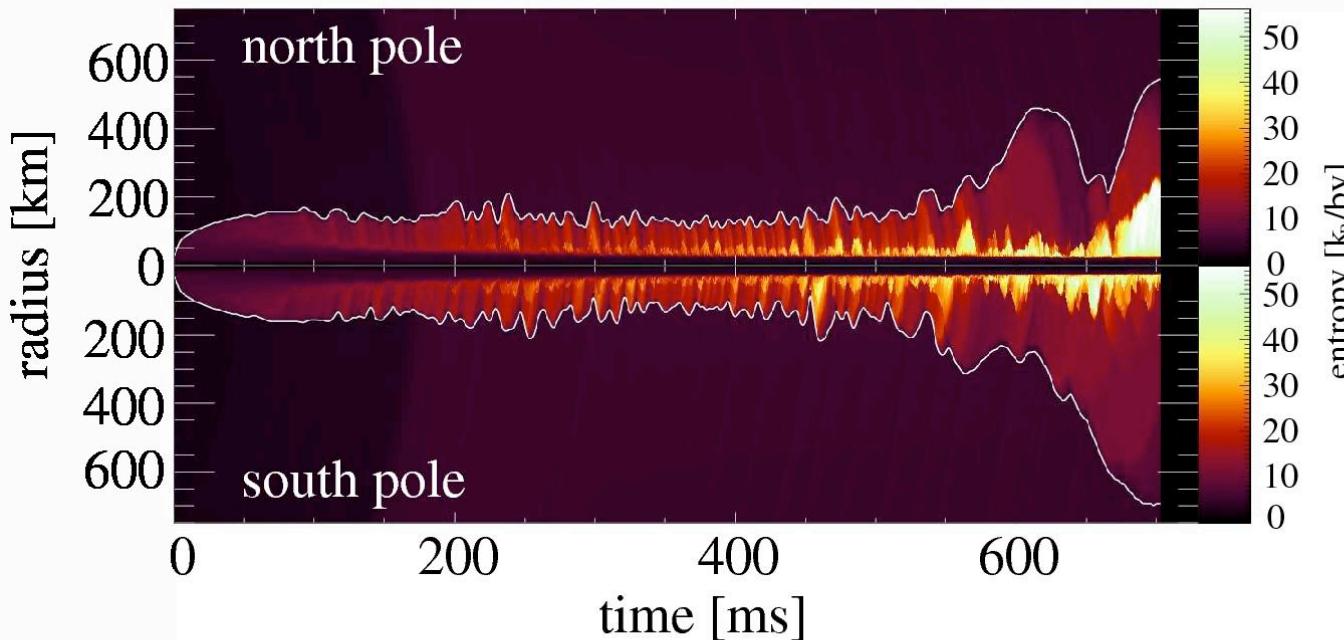
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# 2D Models in axisymmetry



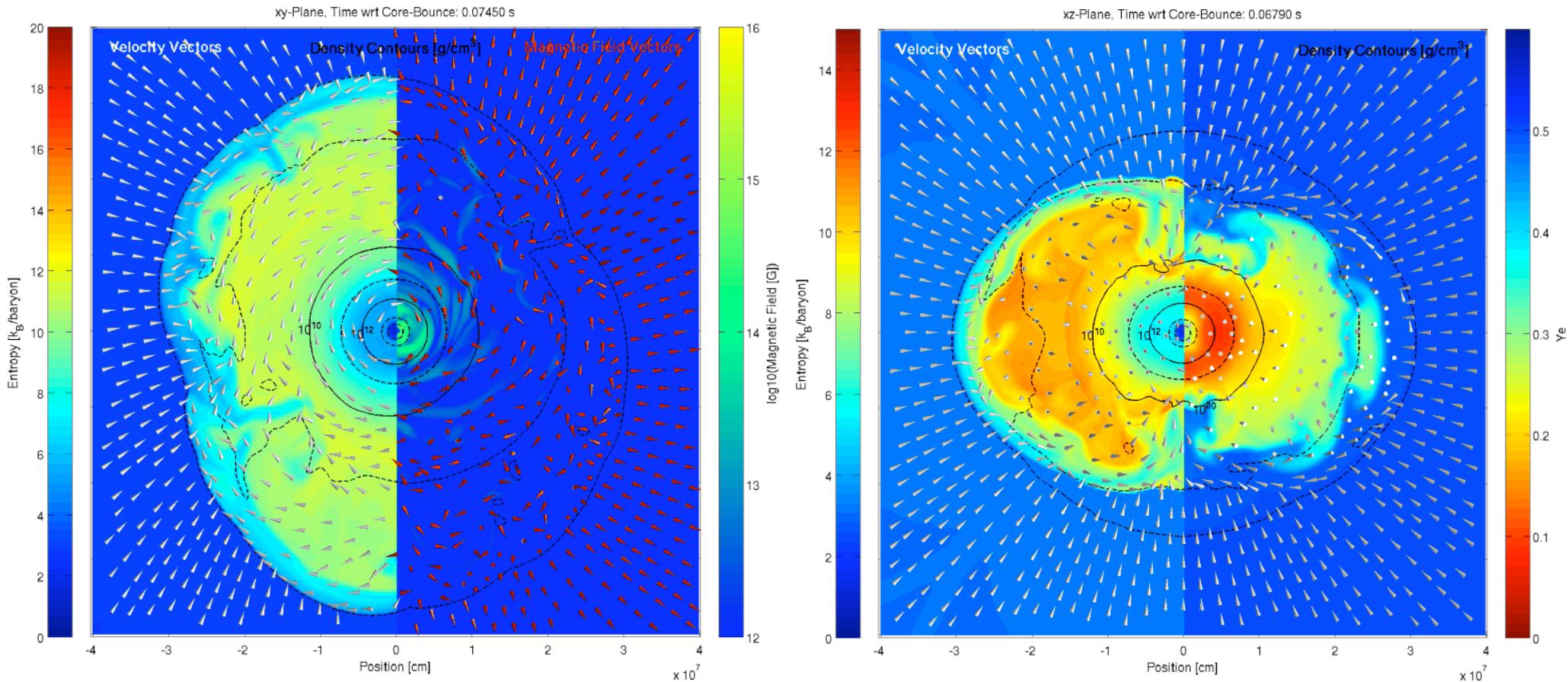
Marek & Janka 2009

- Spectral neutrino transport
- Ray-by-ray approach couples transport in angular wedges to 2D hydrodynamics

## Talks by

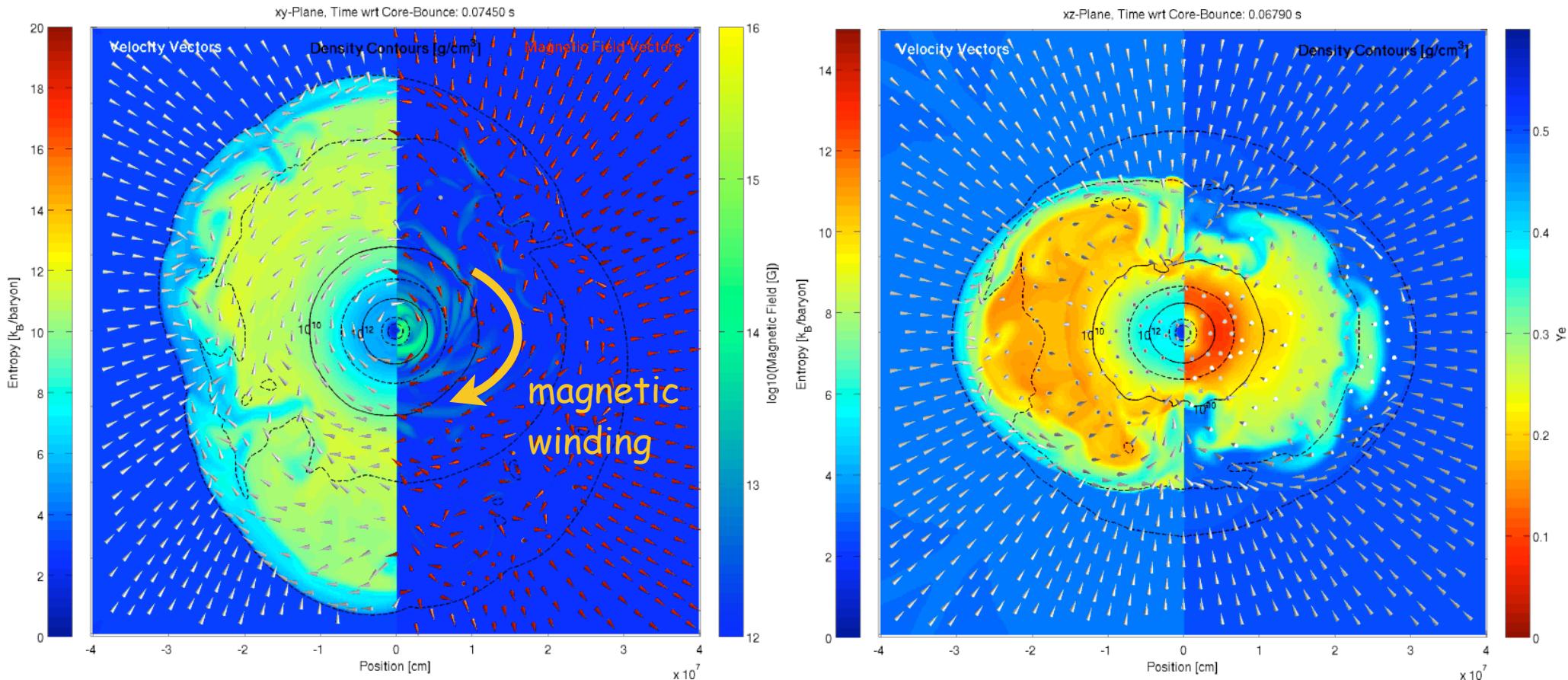
- Steve Bruenn
- Christian Ott
- Bernhard Müller
- Thomas Janka
- Neutrino heating
- Standing Accretion Shock Instability
- Input physics
- Asymmetries
- Grid/Coordinates
- Relativity /  $O(v/c)$

# FISH code: 3D magneto-hydrodynamics



- Convective turnover in 2D is restricted to toroidal shapes!
- Tube-shaped downstreams and broad upflows cannot be modelled in 2D.
- Fluid instabilities and coupling to magnetic are intrinsically 3D.

# FISH code: 3D magneto-hydrodynamics



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- Fluid instabilities and coupling to magnetic are intrinsically 3D.

# Solving the Boltzmann equation



$$\begin{aligned}
 & \frac{\partial F}{\alpha c \partial t} + \frac{\partial (4\pi r^2 \alpha \rho \mu F)}{\alpha \partial m} + \Gamma \left( \frac{1}{r} - \frac{\partial \alpha}{\alpha \partial r} \right) \frac{\partial [(1 - \mu^2) F]}{\partial \mu} \\
 & + \left( \frac{\partial \ln \rho}{\alpha c \partial t} + \frac{3u}{rc} \right) \frac{\partial [\mu (1 - \mu^2) F]}{\partial \mu} \\
 & + \left[ \mu^2 \left( \frac{\partial \ln \rho}{\alpha c \partial t} + \frac{3u}{rc} \right) - \frac{1}{rc} u - \mu \Gamma \frac{\partial \alpha}{\alpha \partial r} \right] \frac{1}{E^2} \frac{\partial (E^3 F)}{\partial E} \\
 & = \frac{j}{\rho} - \tilde{\chi} F + \frac{1}{h^3 c^4} E^2 \int d\mu' R_{is}(\mu, \mu', E) F(\mu', E) \\
 & - \frac{1}{h^3 c^4} E^2 F \int d\mu' R_{is}(\mu, \mu', E) \\
 & + \frac{1}{h^3 c^4} \left[ \frac{1}{\rho} - F(\mu, E) \right] \int E'^2 dE' d\mu' \tilde{R}_{nes}^{in}(\mu, \mu', E, E') F(\mu', E) \\
 & - \frac{1}{h^3 c^4} F(\mu, E) \int E'^2 dE' d\mu' \tilde{R}_{nes}^{out}(\mu, \mu', E, E') \left[ \frac{1}{\rho} - F(\mu', E') \right]
 \end{aligned}$$

$$\frac{\partial Y_e}{\partial t} = -\frac{2\pi m_B}{h^3 c^2} \int E^2 dE d\mu \left( \frac{j}{\rho} - \tilde{\chi} F \right) \quad \frac{\partial e}{\partial t} = \dots \quad \frac{\partial u}{\partial t} = \dots$$

(Mezzacappa & Bruenn 1993, Liebendörfer 2000, Liebendörfer et al. 2004)

Evolution of specific neutrino distr. function:

$$F(t, m, \mu, E) = f(t, r, \mu, E) / \rho$$

=> 3D implicit problem

Comoving metric:

$$\begin{aligned}
 ds^2 &= -\alpha^2 dt^2 + \left( \frac{1}{\Gamma} \frac{\partial r}{\partial a} \right)^2 \\
 &+ r^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2)
 \end{aligned}$$

Stress-energy tensor:

$$\begin{aligned}
 T^{tt} &= \rho (1 + e + J) \\
 T^{ta} = T^{at} &= \rho H \\
 T^{aa} &= p + \rho K \\
 T^{\vartheta\vartheta} = T^{\varphi\varphi} &= p + \frac{1}{2} \rho (J - K)
 \end{aligned}$$

# Pitfalls of multi-D Boltzmann $\nu$ -transport



## Boltzmann transport:

- One fluid element contains  
4  $\nu$  types  $\times$  20 energies  $\times$  100 angles = 8000 variables
- At a resolution of  $1000^3$  zones  
--> 64TB per time step

## Hydrodynamics:

- One fluid element  
contains ~10 variables
- At a resolution of  
 $1000^3$  zones  
--> 80GB per step

# Pitfalls of multi-D Boltzmann $v$ -transport

Boltzmann transport:

- One fluid element contains  
 $4 v \text{ types} \times 20 \text{ energies} \times 100 \text{ angles} = 8000 \text{ variables}$
- At a resolution of  $1000^3$  zones  
 $\rightarrow 64\text{TB per time step}$

Compression of Fermi-gas:

$$\frac{dF}{dt} - \frac{1}{3E^2} \frac{\partial}{\partial E} (E^3 \rho F) \frac{d}{dt} \left( \frac{1}{\rho} \right) - \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{c\lambda}{3} \frac{\partial F}{\partial r} \right) = \left( \frac{dF}{dt} \right)_{\text{collision}}$$

$de$        $p dV$       diffusion      = interactions

Hydrodynamics:

- One fluid element contains  $\sim 10$  variables
- At a resolution of  $1000^3$  zones  
 $\rightarrow 80\text{GB per step}$

difficult energy-terms  
 must not be neglected!

# Pitfalls of multi-D Boltzmann $v$ -transport

Boltzmann transport:

- One fluid element contains  
 $4 v \text{ types} \times 20 \text{ energies} \times 100 \text{ angles} = 8000 \text{ variables}$
- At a resolution of  $1000^3$  zones  
 $\rightarrow 64\text{TB per time step}$

Compression of Fermi-gas:

$$\frac{dF}{dt} - \frac{1}{3E^2} \frac{\partial}{\partial E} (E^3 \rho F) \frac{d}{dt} \left( \frac{1}{\rho} \right) - \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{c\lambda}{3} \frac{\partial F}{\partial r} \right) = \left( \frac{dF}{dt} \right)_{\text{collision}}$$

$de$        $p dV$       diffusion      = interactions

Diffusion limit:

$$\frac{\lambda}{3} \frac{\partial F}{\partial r} \ll F, \quad \frac{H}{cJ} \sim 10^{-4}, \quad H = \int_{-1}^{+1} F(\mu) \mu d\mu$$

Hydrodynamics:

- One fluid element contains  $\sim 10$  variables
- At a resolution of  $1000^3$  zones  
 $\rightarrow 80\text{GB per step}$

difficult energy-terms  
 must not be neglected!

Inaccurate fluxes in  
 diffusion-regime due to  
 large cancellations in  
 angle integral!

# There is no perfect transport algorithm...

Diffusive regime	Semi-transparent	Transparent regime
Boltzmann solver	Truncation errors in flux	Inefficient ang. resol.
Flux-limited diffusion		Flux-factor estimated
Ray-tracing	Short mean free path	Limited by reaction rates

The ideal algorithm combines the three green fields!  
 However, it might be too complicated. Alternatives:

Interpolate from diffusive regime to transparent regime:

- Multi-Group flux-limited diffusion  
 Flux factor unknown in transparent regime!
- New: Isotropic diff. source approximation  
 $f = f(\text{trapped}) + f(\text{streaming}),$   
 separate evolution Eqs.

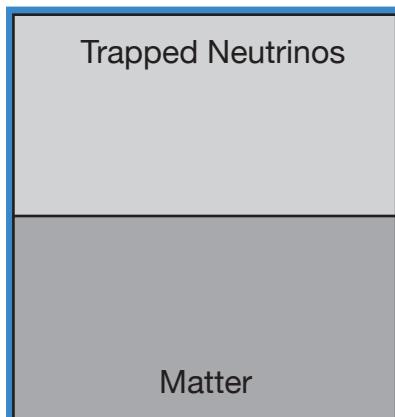
# Isotropic Diffusion Source Approximation

$$D(f) = j - \chi^* f$$

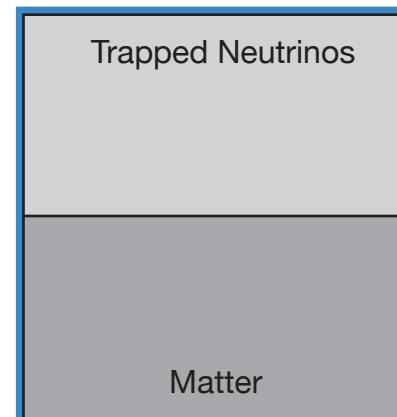
$$f = f(\text{trapped}) + f(\text{streaming}) = f_t + f_s$$

Different approx.  
for trapped & streaming  
neutrino components!

Fluid element A



Fluid element B



Streaming Neutrinos

Liebendörfer et al. 2009

# Isotropic Diffusion Source Approximation

$$D(f) = j - \chi^* f$$

$$f = f(\text{trapped}) + f(\text{streaming}) = f_t + f_s$$

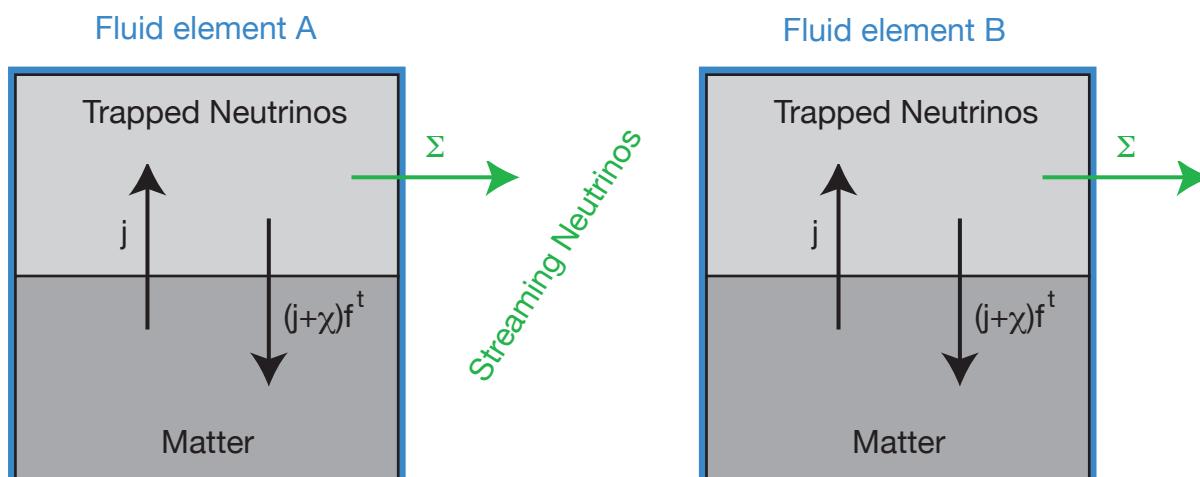
$$D(f_t) = j - \chi^* f_t - \Sigma \quad (1)$$

$$D(f_s) = -\chi^* f_s + \Sigma \quad (2)$$

Different approx.  
for trapped & streaming  
neutrino components!

$\Sigma$  determined by diffusion limit of (1)

- implicit local weak interaction solver
- advection-diffusion problem



# Isotropic Diffusion Source Approximation

$$D(f) = j - \chi^* f$$

$$f = f(\text{trapped}) + f(\text{streaming}) = f_t + f_s$$

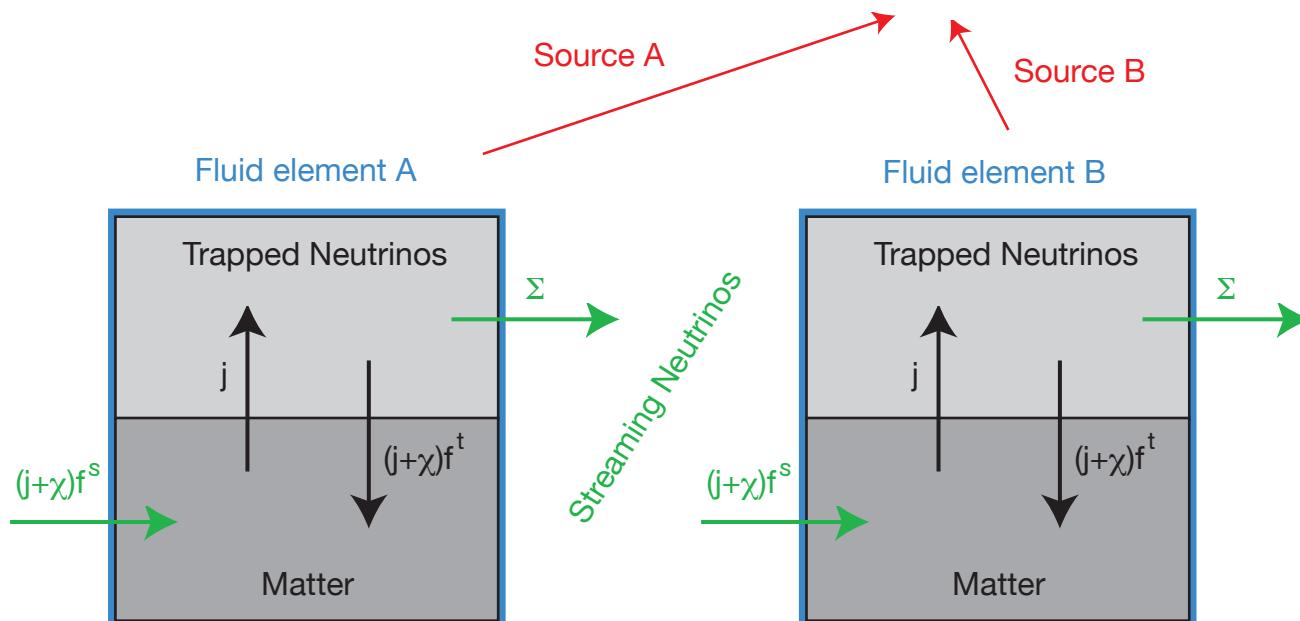
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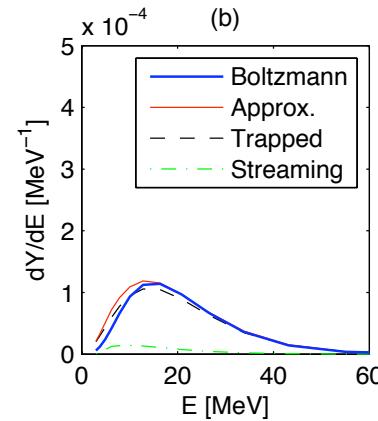
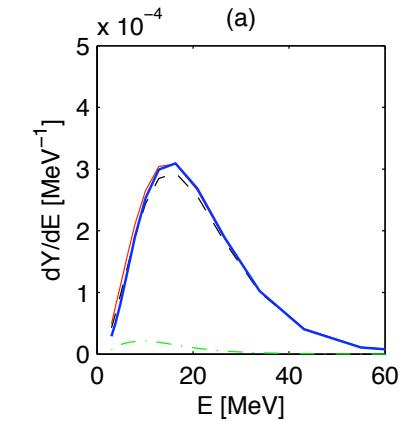
$\Sigma$  determined by diffusion limit of (1)

Stationary state approx. for (2) --> Poisson Eq.

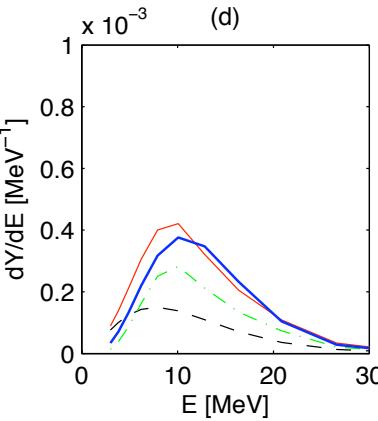
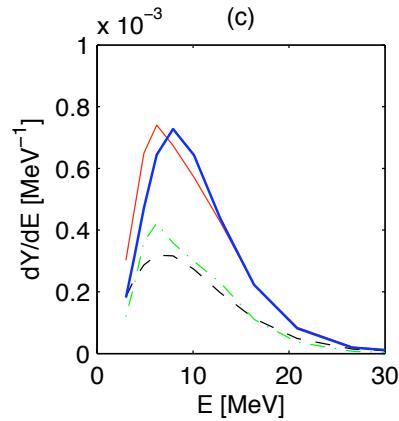


- implicit local weak interaction solver
- advection-diffusion problem
- 20+1 Poisson solves per step
- Geometrical analysis of neutrino-spheres

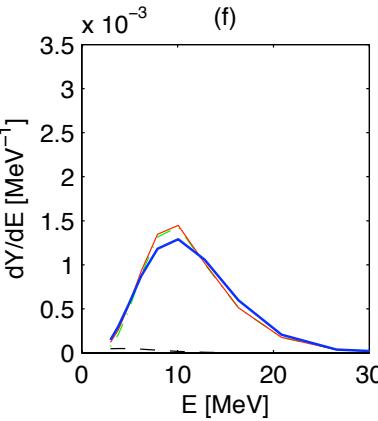
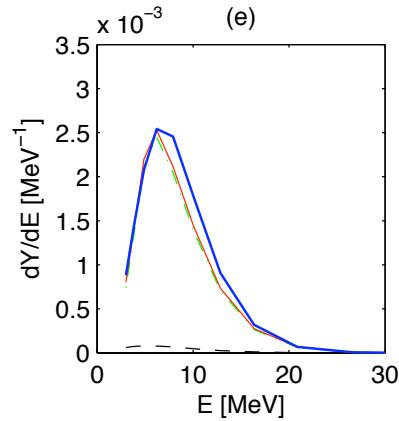
# Comparison of IDSA Spectra



at 40 km radius  
(trapped regime)



at 80 km radius  
(semi-transparent)



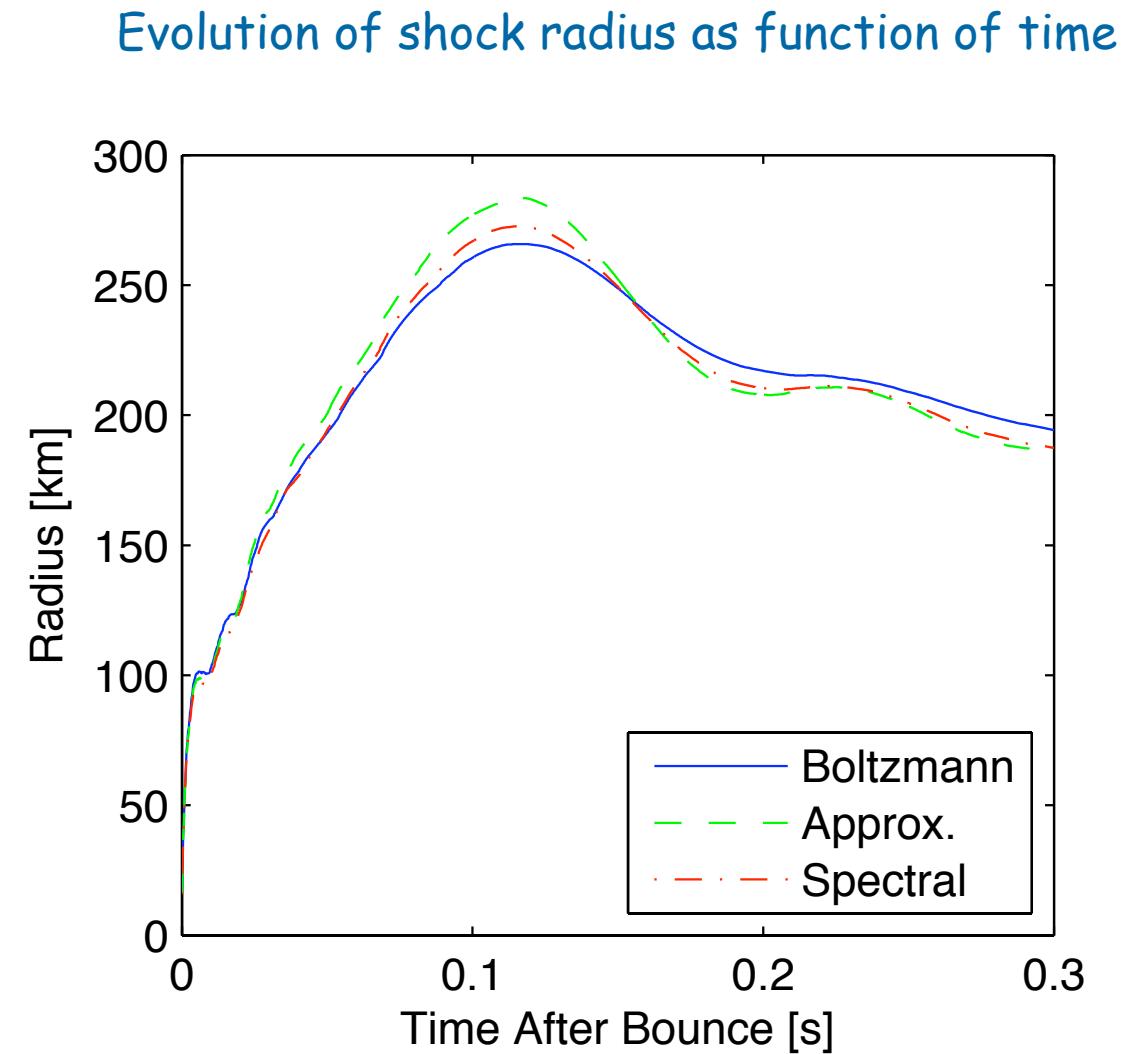
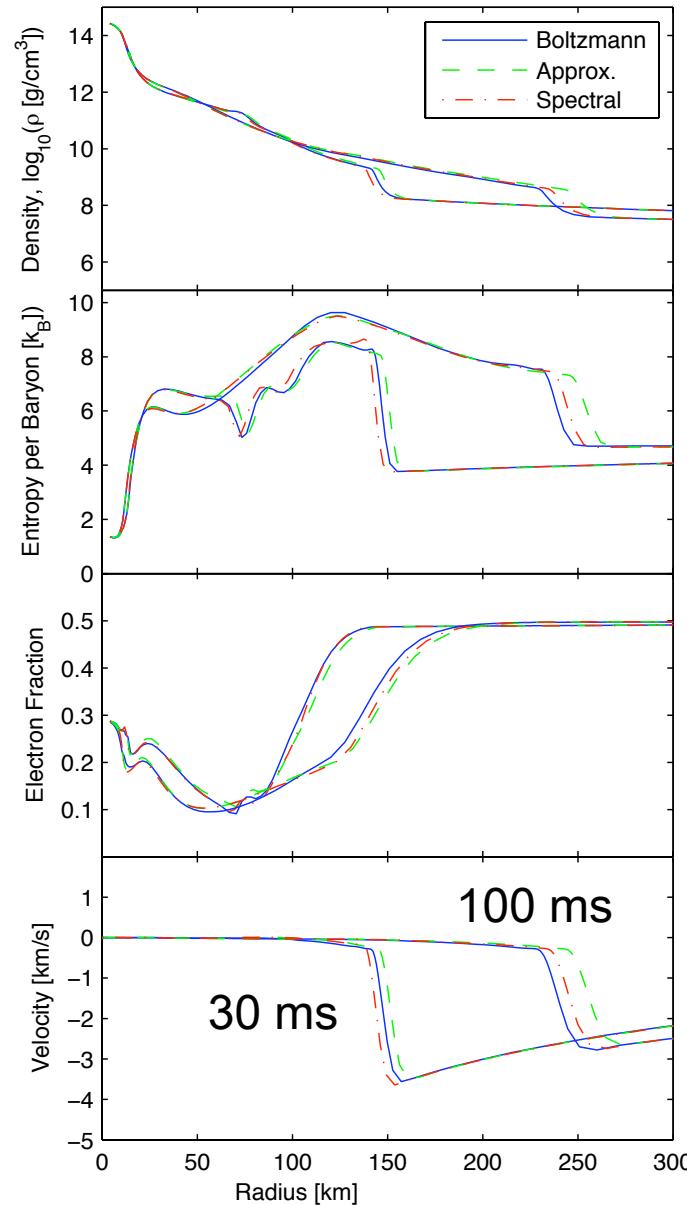
at 160 km radius  
(free streaming)

Trapped neutrinos  
dominate spectrum

Trapped \*and\*  
streaming neutrinos  
form spectrum

Streaming neutrinos  
dominate spectrum

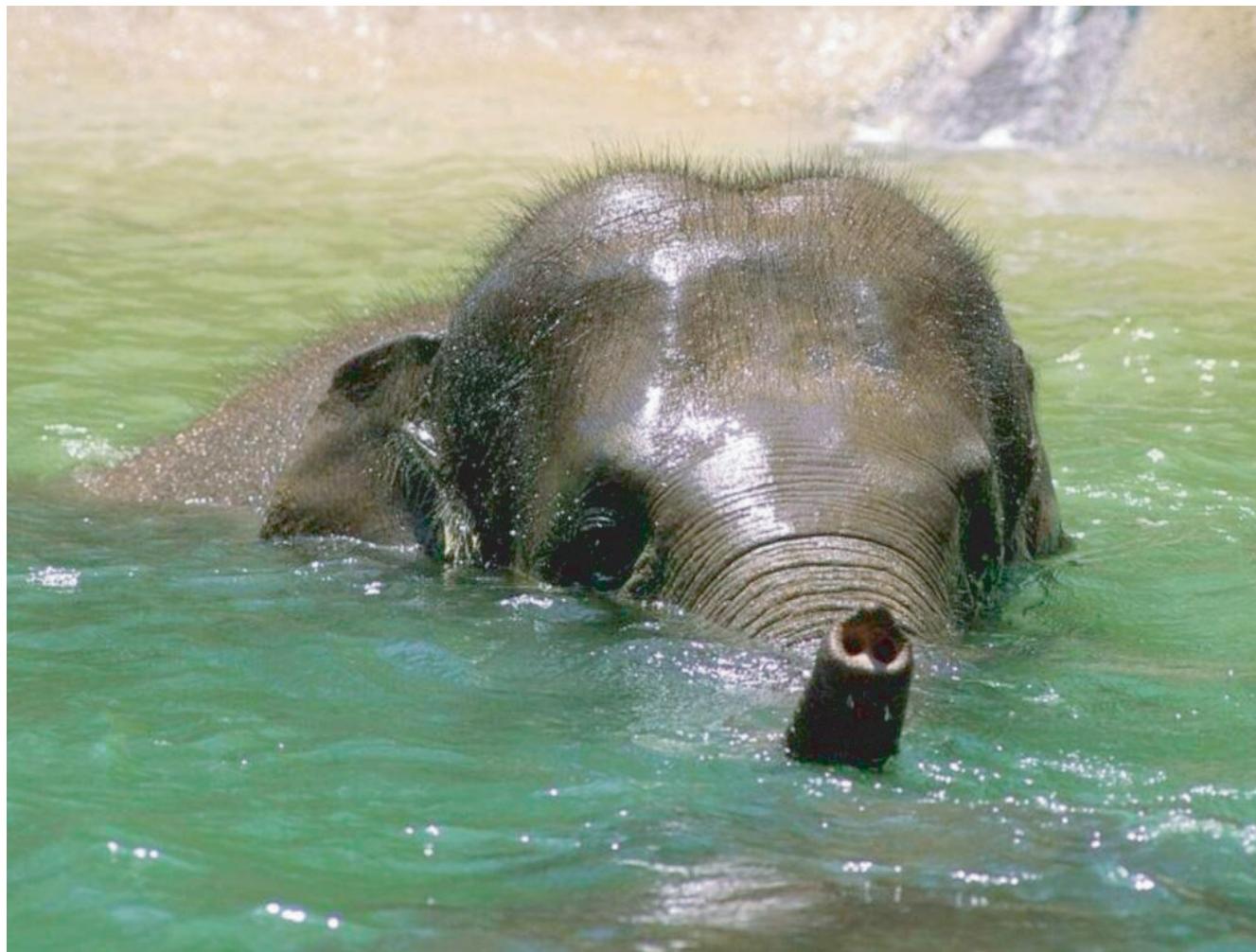
# Comparison of Hydrodynamical Evolution



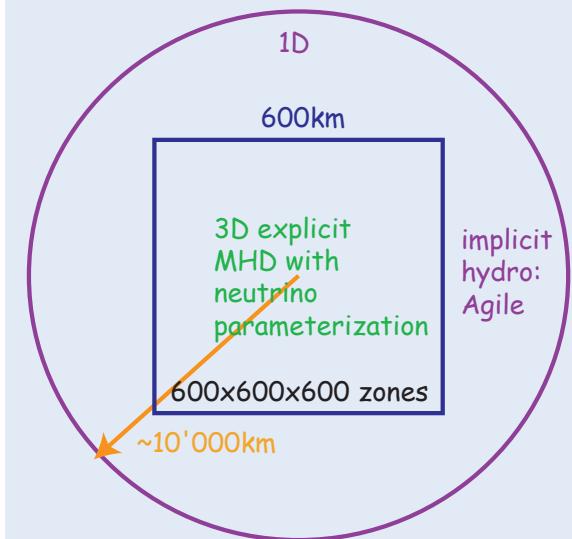
prel. Agile & IDSA available by email  
 to matthias.liebendoerfer at unibas.ch

# From FISH to ELEPHANT...

Elegant parallel hydrodynamics with  
approximate neutrino transport

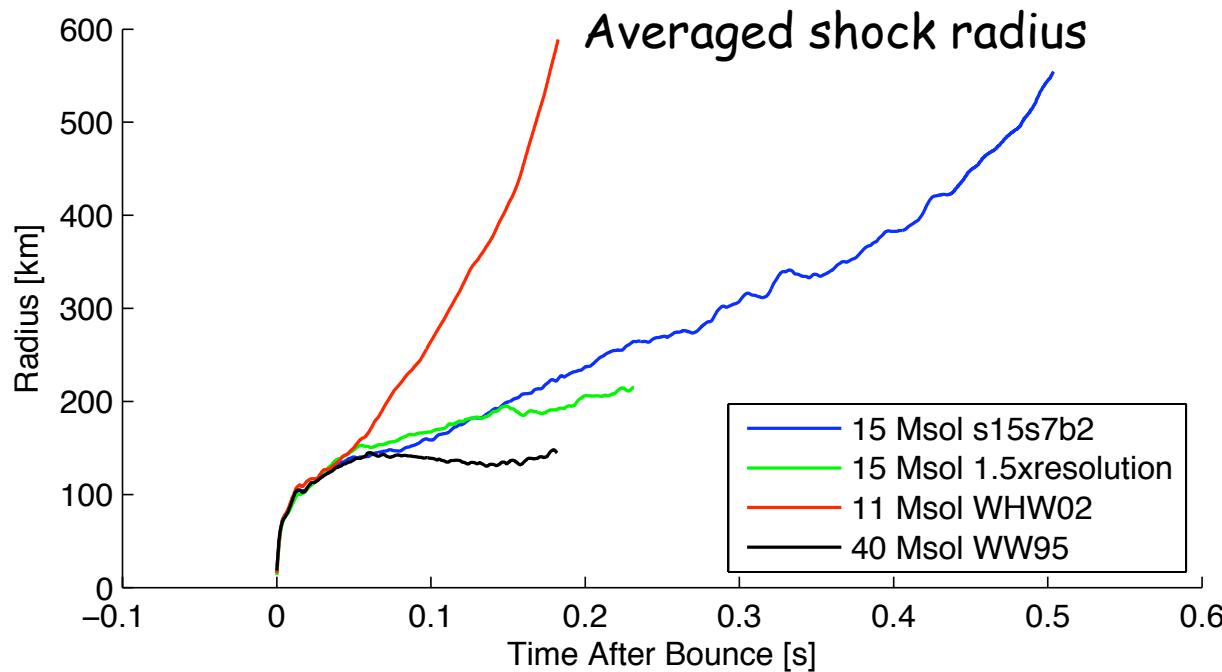


- Lattimer-Swesty EoS
- Effective GR potential
- constrained  $\text{div}(\mathbf{B})$
- 2nd order TVD
- IDSA for e-flavour  $\nu$ 's
- Leakage for  $\mu/\tau$   $\nu$ 's



(Whitehouse/Käppeli et al.)

# 3D supernova model with IDSA



- The 11 solar mass progenitor run shows positive velocities and produces an explosion
- The 15 solar mass progenitor has not (yet) developed positive velocities, even if the shock radius increases.

Runs more optimistic than (Marek & Janka 2009) 2D models.

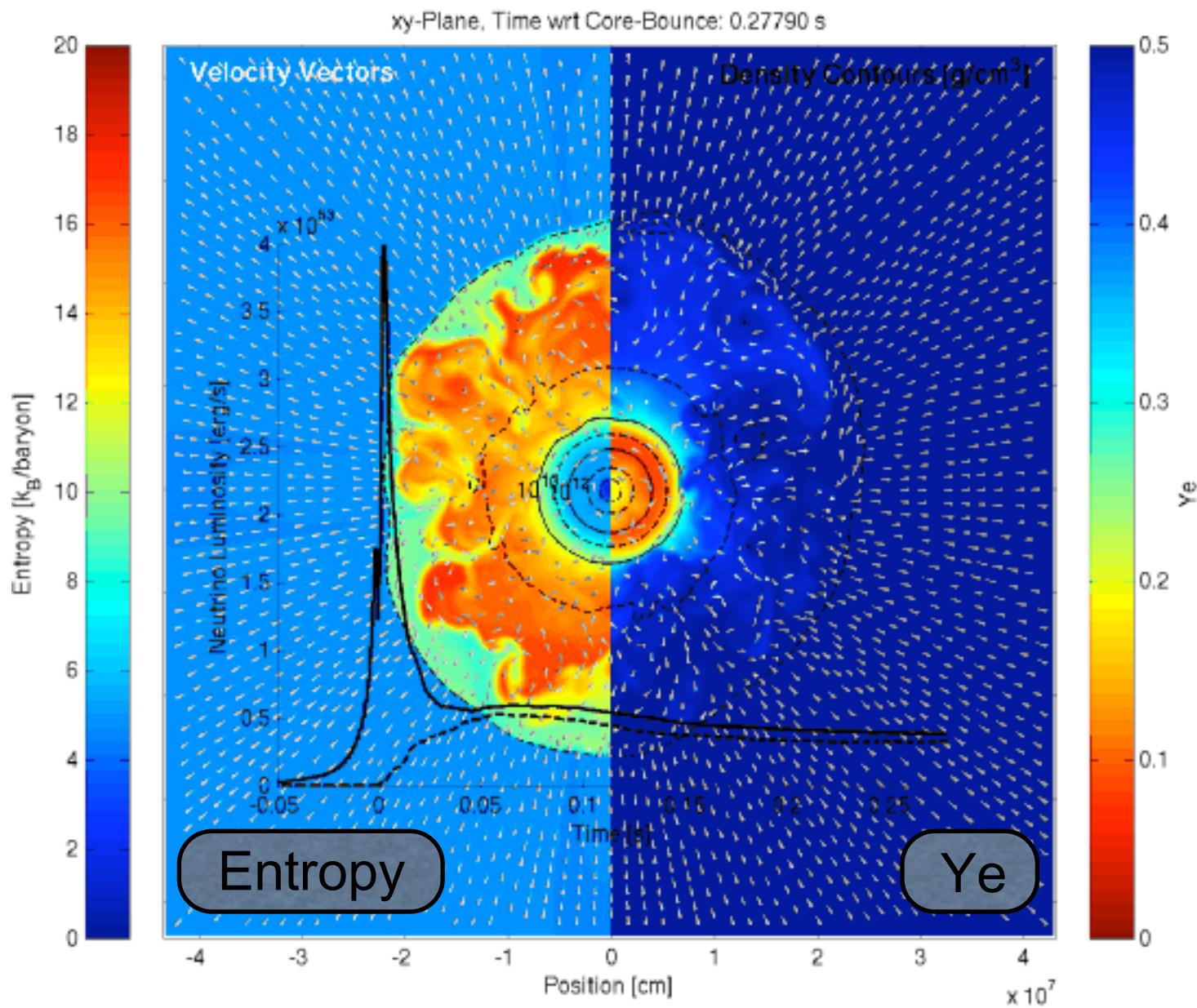
Runs more pessimistic than (Hix et al. 2010) 2D models.

Convergence test:  
blue run perhaps still too optimistic?

Entropy

Ye

3D Elephant code, PoS(NIC X)243, arXiv:0711.2929, arXiv:0910.2854 (s15s7b2, red profiles ->)



Comparison with 1D GR Boltzmann v transport: ApJ 620 (2005) 840, (Model G15, blue profiles ->)

