Turbulence and Supernova Neutrinos

Jim Kneller
NC State

The signal from the next Galactic supernova

The neutrinos we will detect from the next SN in our Galaxy will tell us much about the explosion *and* the neutrino.

The flavour content of the signal changes as it propagates from the proto-neutron star to our detectors here on Earth.

It evolves a number of times during the voyage:

- within the first 1000 km due to neutrino self interactions,
- due to the effect of matter (MSW),
  - with the added effect of turbulence,
- de-coherence as the neutrino propagates to Earth,
- and then Earth matter effects if the SN is 'shadowed'.

Turbulence (density fluctuations) is not yet satisfactorily included in calculations of the expected neutrino signal.

Turbulence may remove spectral features that indicate the hierarchy, θ_{13} etc. and do not allow us to peer inside the SN.

Sawyer, PRD, **42** 3908 (1990)

Loreti *et al.*, PRD, **52**, 6664 (1995)

Fogli *et al.*, JCAP, **0606**, 012 (2006)

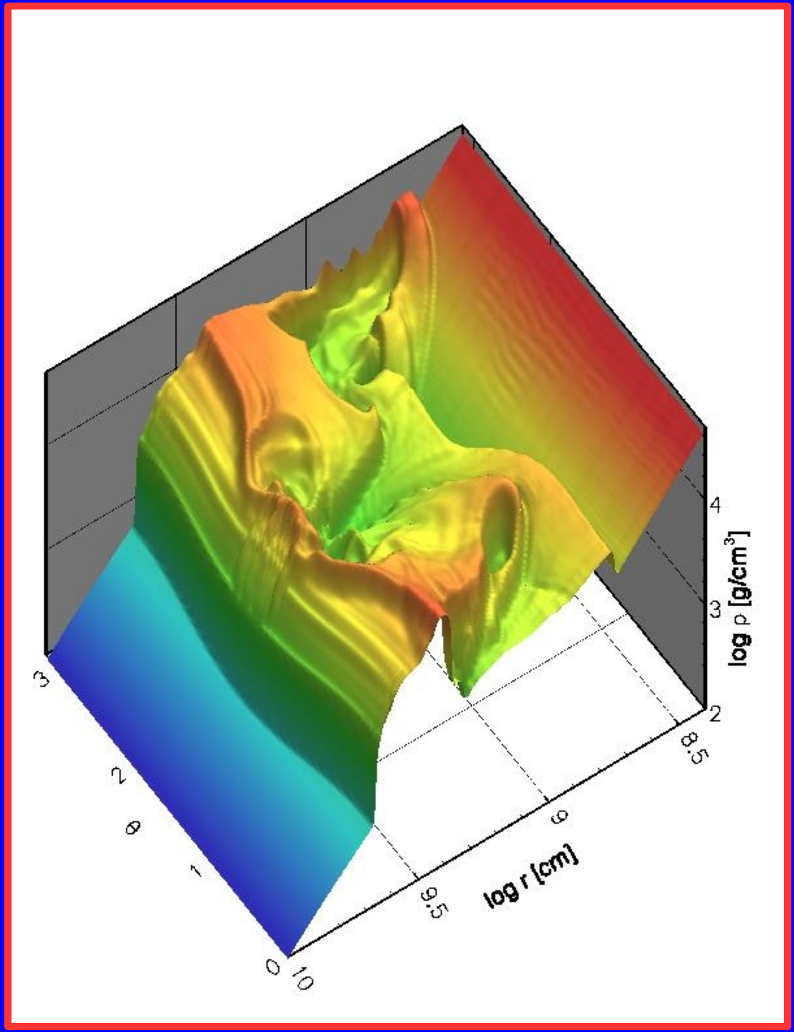
Friedland & Gruzinov, arXiv:astro-ph/0607244

A better understanding of turbulence effects and the implications for observables would be very desirable.

Kneller & Volpe, PRD **82** 123004 (2010)
Kneller, arXiv:1004.1288 (2010)

Turbulence in supernova

In multi-d hydro simulations we see turbulence generated by aspherical flows through distorted shocks, convection, etc.



Kneller, McLaughlin & Brockman,
PRD 77 045023 (2008)

We take a supernova profile from a 1D hydrodynamical supernova simulation and add turbulence to it i.e.

$$V_e(r) = (1 + F(r)) \langle V_e(r) \rangle$$

where *F(r)* is a Gaussian random field with rms amplitude C_*.

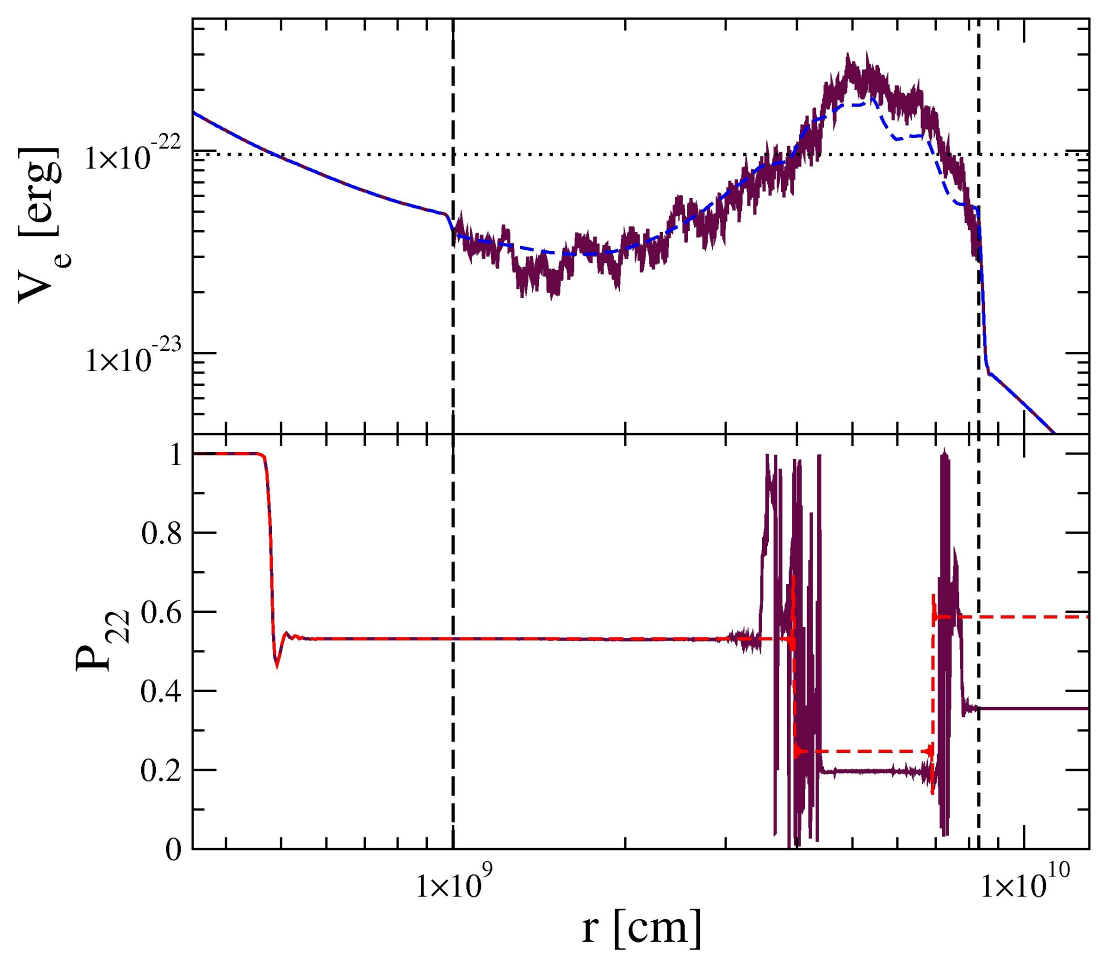
The turbulence is restricted to the region between the shocks.

The ν state at r is related to the initial state through an operator *S*. The probability that an initial state *j* is later detected as state *i* is

$$P(\nu_j \rightarrow \nu_i) = |S_{ij}|^2$$

Our approach is to generate realizations *F* so as to construct an ensemble of S and P_{ij}'s.

For example:

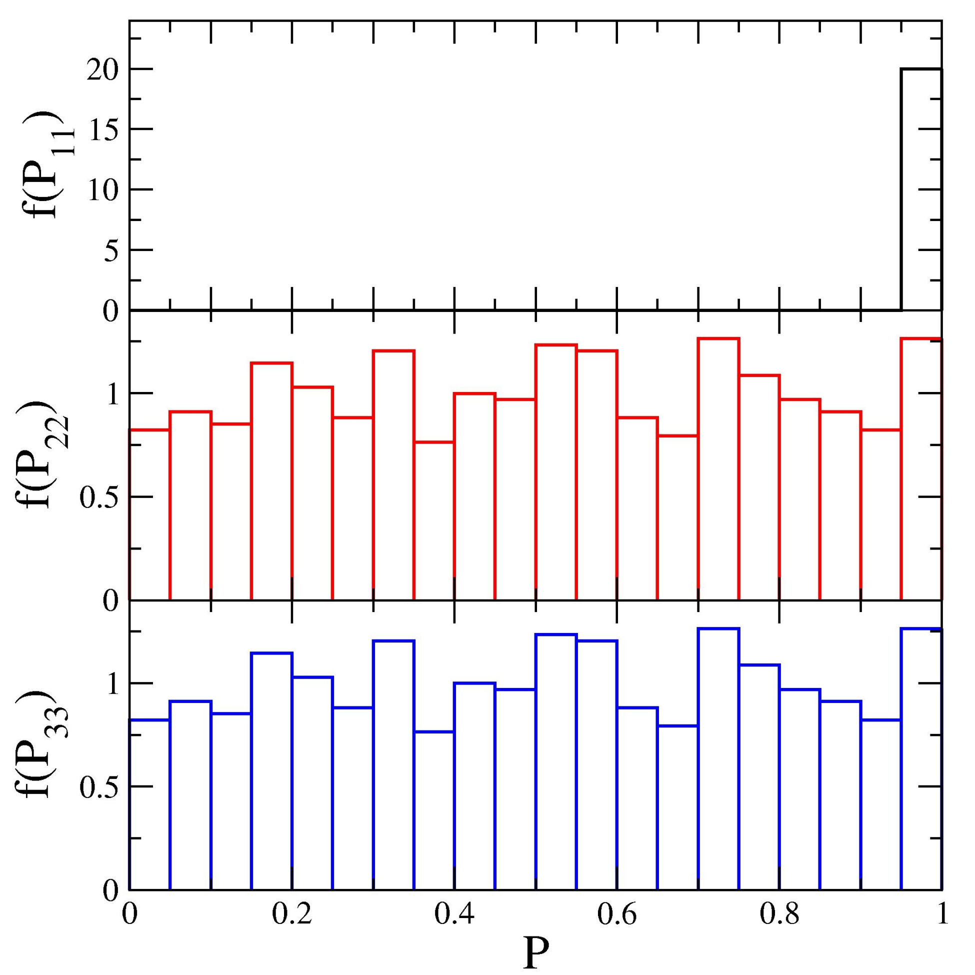


NH, E = 25 MeV, C_* = 0.32, $\sin^2 2\theta_{13} = 4 \times 10^{-4}$

Turbulence introduces many more MSW resonances and also one observes off-resonance evolution.

Small amplitudes and the H resonance channel

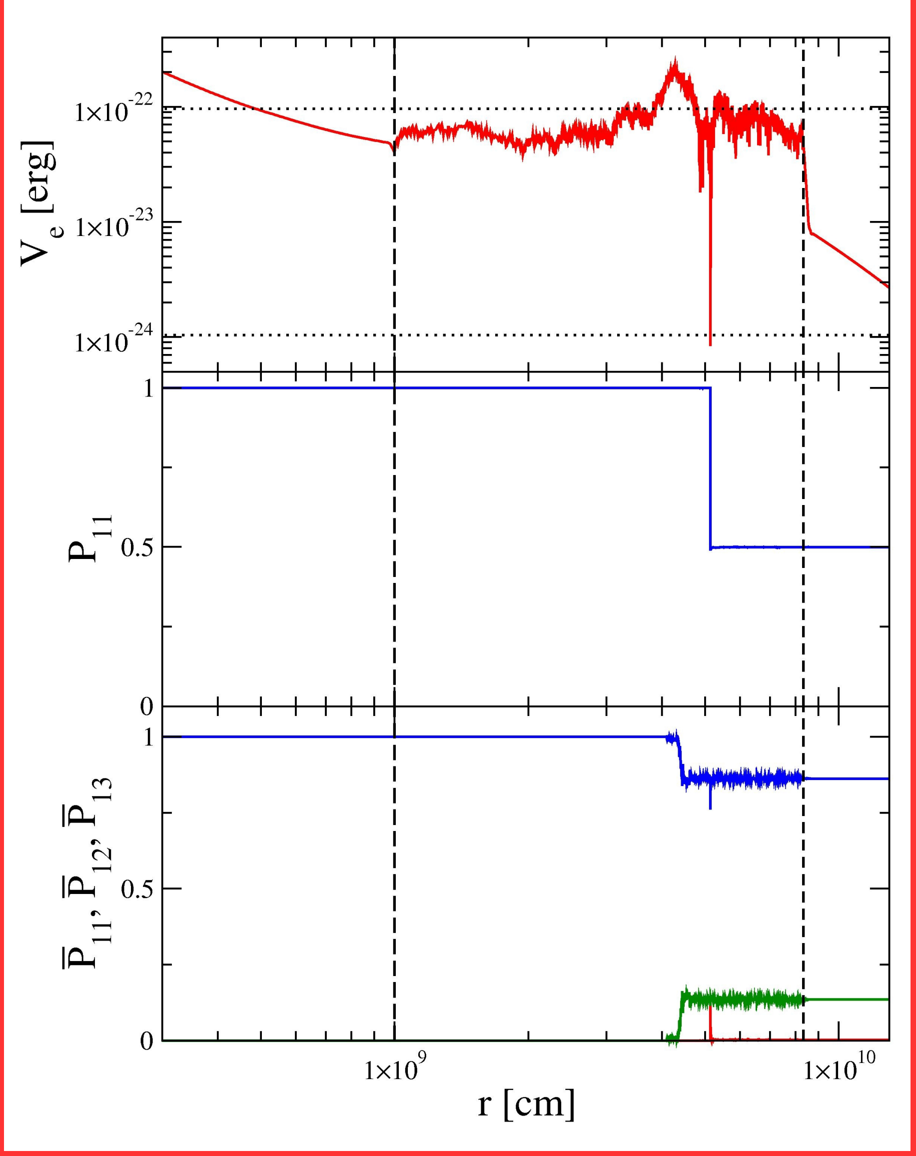
Final state distribution from >1000 realizations of the turbulence.



NH, E = 25 MeV, $C_* = 0.1$, $\sin^2 2\theta_{13} = 4 \times 10^{-4}$

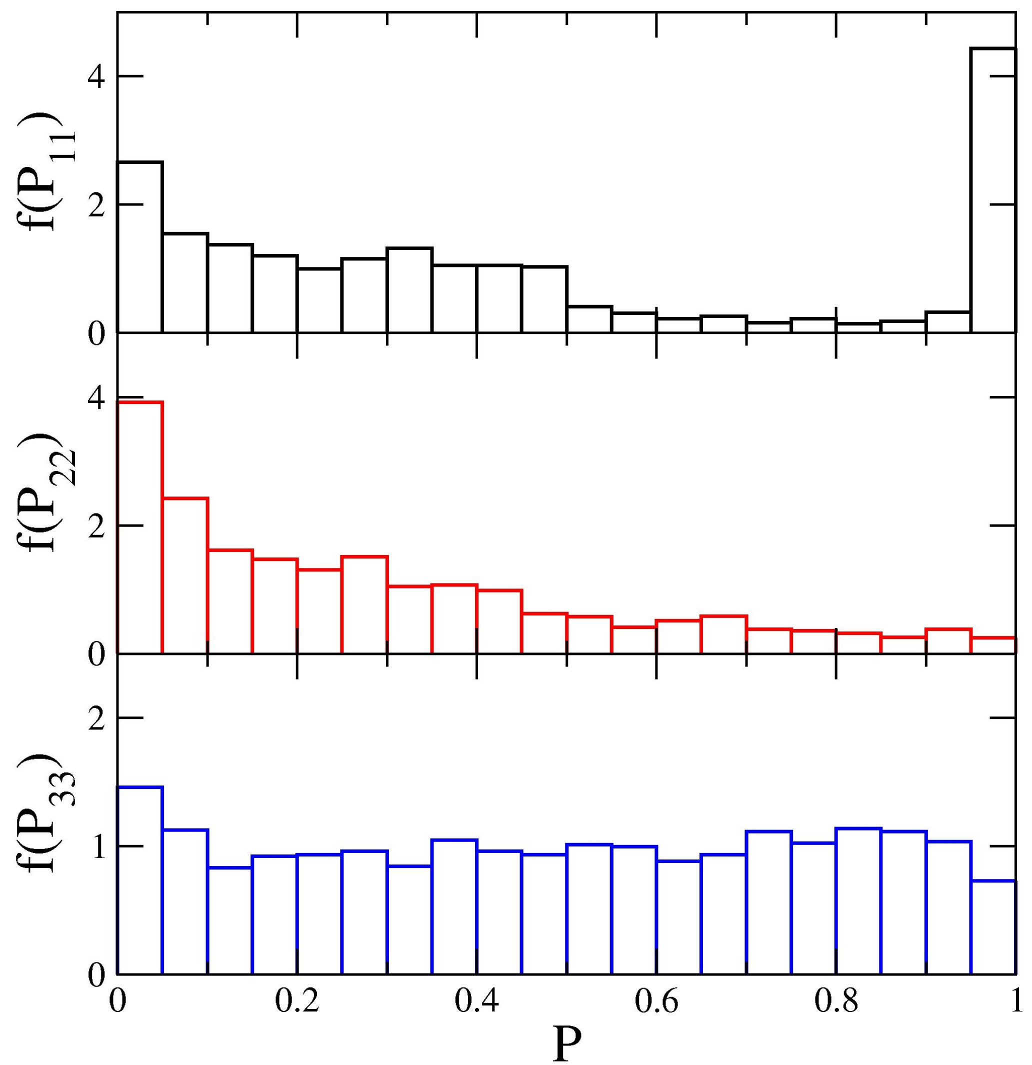
Normal hierarchy and three flavour effects

For large amplitudes we break HL factorization.



NH, E = 25 MeV, $C_* = 0.32$, $\theta_{13} = 9°$

The distributions of the final state probabilities become triangular.



NH, E = 60 MeV, $C_* = 0.55$, $\theta_{13} = 9°$

Products of random unitary matrices

S-matrices can be factored:

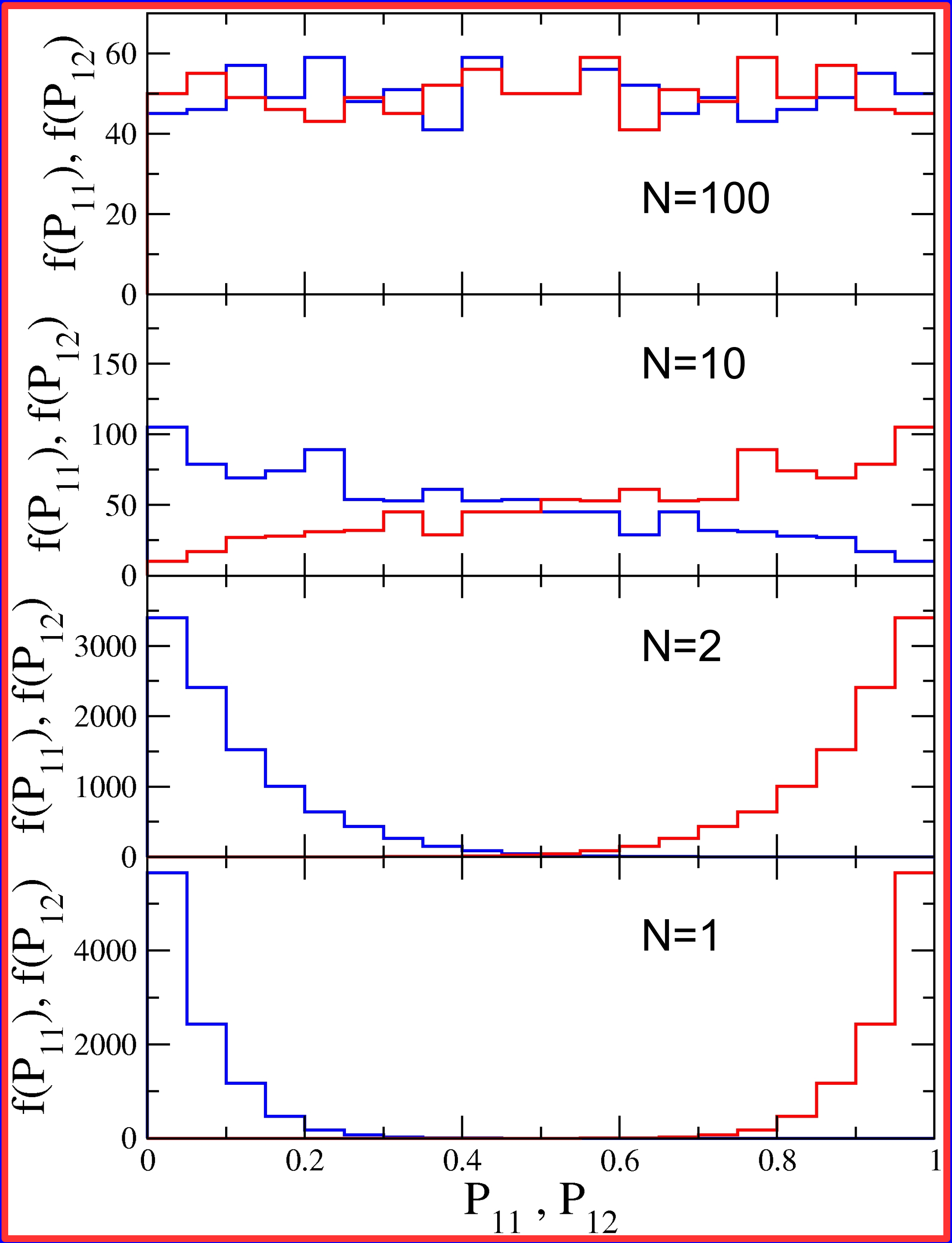
$$S(X,0)=S(X,x_N)\ldots S(x_1,0)$$

We can think about breaking the integration domain into N subdomains each with one resonance.

Each domain is described by a S-matrix that we could regard as a random matrix.

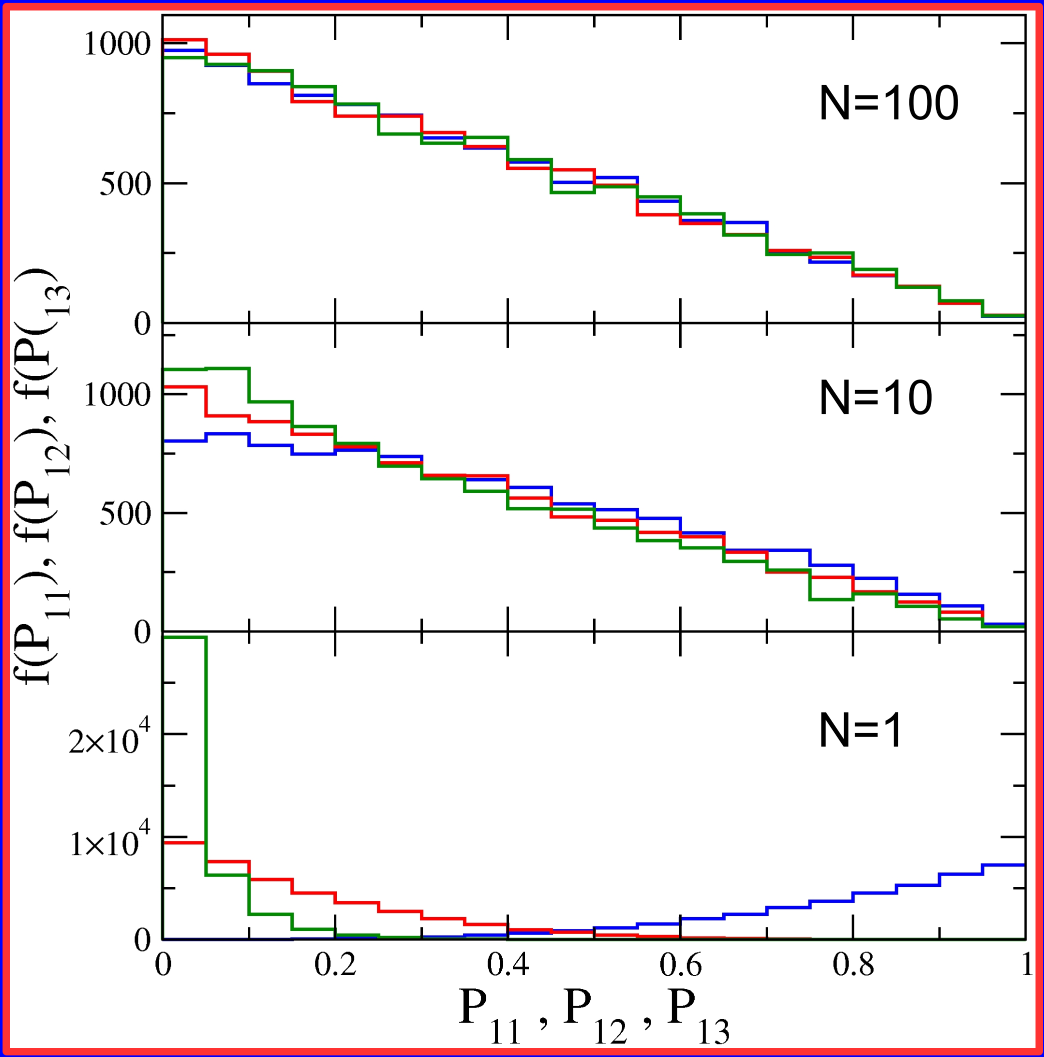
For some given, non-circular, distribution of the S-matrices in each subdomain, what are the properties of the S-matrix for the entire domain.

An ensemble of the matrix product of N random, unitary matrix factors is CUE(N_f) as $N \to \infty$ for all distributions of the factors.



For $N_f = 2$ we can start with diagonally dominant matrices and end up with uniform distributions.

For N_f = 3 we get triangular distributions.



N_f flavour depolariazation

Depolarization means that there is no connection between the initial and final states: all final states are equal.

In the depolarized limit the ensemble of S-matrices is a realization of Dyson's Circular Unitary Ensemble CUE(N_f).

The distribution of the set of probabilities {P_{1j}, P_{2j}, ...} for observing final states ν_1, ν_2, … is uniform over a standard N_f-1 simplex.

Integrating over N_f-1 elements of the set one finds that the distribution of probability P_{ij} is

$$P(P_{ij}) = (N_f - 1)(1 - P_{ij})^{(N_f - 2)}$$

Summary

The turbulence features very much depend upon the amplitude and the mixing parameters.

- For small amplitudes:
  - turbulence is quasi two flavour,
  - appears only in the H resonant channel.
- For larger amplitudes:
  - turbulence breaks HL i.e. it is 3 flavour,
  - appears in non-resonant channel.
- The depolarized limit is a function of the number of flavours.

Turbulence in the progenitor